THE UNIVERSITY OF MANITOBA

DATE: December 16, 2013 (Afternoon) **FINAL EXAMINATION**

DEPARTMENT & COURSE NO: MATH2132 **TIME**: 3 hours

EXAMINATION: Engineering Mathematical Analysis 2 **EXAMINER**: D. Trim

6 **1.** Find the open interval of convergence for the series

$$\sum_{n=3}^{\infty} \frac{(n+1)!}{n^n}(x+1)^{4n+3}.$$

14 **2.** Find the Taylor series for the function

$$f(x) = \sqrt{1+3x},$$

about $x = 2$. You must use a method that guarantees that the series converges to the function. Write the series in sigma notation, simplified as much as possible. What is the radius of convergence of the series?

10 **3.** Find the sum of the series

$$\sum_{n=1}^{\infty} (n+1)2^n x^{n-1}.$$

12 **4.** Solve the initial-value problem

$$y'' = yy', \quad y(0) = 1, \quad y'(0) = 1/2.$$

(This is from Section 15.4 which we omitted.)

6 **5.** Find the form of a particular solution of the differential equation

$$D(D^2-1)(D^2+4)y = 3x^2e^{-x} + 10x + 5\cos x$$

as predicted by the method of undetermined coefficients. Do **NOT** solve for the coefficients.

12 **6.** A 100 gram mass is suspended from a spring with constant 25/2 newtons per metre. It is set into vertical motion by pulling it 5 centimetres below its equilibrium position and giving it velocity 2 metres per second upward. During its subsequent motion, damping is equal to 3 times velocity.

(a) Find the position of the mass as a function of time.

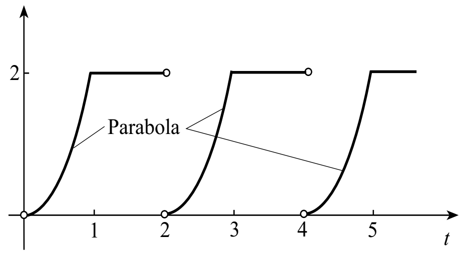
(b) Is the motion underdamped, overdamped, or critically damped?

(c) Determine whether the mass ever passes through its equilibrium position. If it does, find the time(s) when this occurs.

8 **7.** Find the Laplace transform of the function

$$f(t) = e^{4t} \sin 3t \, h(t-2).$$

10 **8.** Find the Laplace transform of the function in the figure below. You need not simplify your result.



10 **9.** Find the inverse Laplace transform for the function

$$\frac{(3s^2 + s - 6)e^{-2s}}{s^3 + 3s^2}.$$

12 **10.** Solve the initial-value problem

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 6y = 4\delta(t-3), \quad y(0) = 2, \quad y'(0) = 1.$$

Answers

1. $-1 - e^{1/4} < x < -1 + e^{1/4}$ **2.** $\sqrt{7} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3^n (2n-2)!}{2^{2n-1} 7^{n-1/2} n!(n-1)!}(x-2)^n$, $7/3$

3. $\frac{4(1-x)}{(1-2x)^2}$, $-1/2 < x < 1/2$

5. $y_p(x) = Ax^3e^{-x} + Bx^2e^{-x} + Cxe^{-x} + Dx^2 + Ex + F\cos x + G\sin x$

6.(a) $x(t) = \frac{3}{80}e^{-5t} - \frac{7}{80}e^{-25t}$ m (b) Overdamped (c) $t = (1/20)\ln(7/3)$ s

7. $e^{8-2s}\left[\frac{3\cos 6}{(s-4)^2+9} + \frac{(s-4)\sin 6}{(s-4)^2+9}\right]$

8. $\frac{1}{1-e^{-2s}}\left[\frac{4}{s^3} - 2e^{-s}\left(\frac{2}{s^3} + \frac{2}{s^2}\right) - \frac{2}{s}e^{-2s}\right]$

9. $(5 - 2t + 2e^{6-3t})h(t-2)$

10. $y(t) = e^{-2t}\left[2\cos\sqrt{2}t + \frac{5}{\sqrt{2}}\sin\sqrt{2}t\right] + \frac{4}{\sqrt{2}}e^{6-2t}\sin\sqrt{2}(t-3)h(t-3)$