THE UNIVERSITY OF MANITOBA

DATE: June 16, 2012 **FINAL EXAMINATION**

DEPARTMENT & COURSE NO: MATH2132 **TIME**: 3 hours

EXAMINATION: Engineering Mathematical Analysis 2 **EXAMINER**: D. Trim

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10 **1.** Find the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(-1)^n n}{4^{n+1}}(x-1)^{2n}.$$

14 **2.** Find the Maclaurin series for the function

$$f(x) = \frac{x}{x^2 - x - 2}.$$

Use a method that guarantees that the series converges to $f(x)$. Express your answer in sigma notation, simplified as much as possible. Determine the interval of convergence for the series.

6 **3.** Find a maximum possible error when the function e^{-3x} is approximated by the first three terms in its Maclaurin series on the interval $0 \le x \le 0.2$.

15 **4.** Find a general solution for the differential equation

$$3y''' + 2y'' + 2y' - y = x - e^{-2x}.$$

6 **5.** You are given that the roots of the auxiliary equation associated with the linear, differential equation

$$\phi(D)y = 2xe^{4x} + x^3 - 2 + 3e^{2x}\cos 5x$$

are $m = 0,\ 2 \pm i,\ 2 \pm i,\ \pm 3,\ 4$. Write down the form of a particular solution of the differential equation as predicted by the method of undetermined coefficients. Do **NOT** find the coefficients, just the form of the particular solution.

6 **6.** When a substance such as glucose is administered intravenously into the bloodstream, it is used up by the body at a rate proportional to the amount present at that time. If it is added at a variable rate $R(t)$, where t is time, and A_0 is the amount in the bloodstream when the intravenous feeding begins, set up, but **DO NOT SOLVE**, an initial value problem for the amount of glucose in the bloodstream at any time. Is the differential equation separable?

7 **7.** Find an implicit definition for the solution of the initial value problem

$$y^2\frac{dy}{dx} = (x+1)(y^3+1), \qquad y(0) = 1.$$

9 **8.** Find the Laplace transform for the function

$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ 4 - t, & 2 < t \le 4 \end{cases} \qquad f(t+4) = f(t).$$

Simplify the transform as much as possible.

9 **9.** Find the inverse Laplace transform for the function

$$F(s) = \frac{e^{-2s}(3s^2 + 2)}{s^3 - s^2 + 2}.$$

8 **10.** A mass of 1 kilogram is suspended from a spring with constant 400 newtons per metre. At time $t = 0$, it is at its equilibrium position and is given velocity 2 metres per second upward. During its subsequent motion, it is also subjected to a damping force that (in newtons) is equal to 40 times its velocity (in metres per second). Use Laplace transforms to find the position of the mass as a function of time.

10 **11.** Solve the initial value problem

$$y'' - 3y' - 4y = 3\delta(t-2), \qquad y(0) = 0, \quad y'(0) = 1.$$

Answers

1. $-1 < x < 3$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3}\left[(-1)^n - \frac{1}{2^n}\right] x^n$

3. $(9/2)(0.2)^3$

4. $\displaystyle C_1 e^{x/3} + e^{-x/2}\left[C_2 \cos\frac{\sqrt{3}x}{2} + C_3 \sin\frac{\sqrt{3}x}{2}\right] - x - 2 + \frac{1}{21}e^{-2x}$

5. $Ax^2e^{4x} + Bxe^{4x} + Cx^4 + Dx^3 + Ex^2 + Fx + Ge^{2x}\cos 5x + He^{2x}\sin 5x$

6. $\displaystyle\frac{dA}{dt} = R(t) - kA, \quad A(0) = A_0,$ No

7. $\displaystyle\frac{1}{3}\ln|y^3 + 1| = \frac{x^2}{2} + x + \frac{1}{3}\ln 2$

8. $\displaystyle\frac{1 - e^{-2s}}{s^2(1 + e^{-2s})}$

9. $\left\{e^{-(t-2)} + 2e^{t-2}[\cos(t-2) + \sin(t-2)]\right\} h(t-2)$

10. $2te^{-20t}$ m

11. $\displaystyle\frac{1}{5}e^{4t} - \frac{1}{5}e^{-t} + \frac{3}{5}\left[e^{4(t-2)} - e^{-(t-2)}\right] h(t-2)$