MATH 2132 Tutorial 11

1. Find Laplace transforms for the following functions:

(a) $f(t) = e^{-2t}\cos 4t\, h(t-3)$

(b) $f(t) = \begin{cases} 2t-5, & 0<t<4 \\ t^2, & 4<t<8 \\ 1 & t>8 \end{cases}$

(c) $f(t) = t^2 - 2t + 3, \quad 0<t<2, \qquad f(t+2) = f(t)$

2. Find inverse Laplace transforms for the following functions:

(a) $F(s) = \dfrac{s^2+3}{s^3+2s^2+s}$

(b) $F(s) = \dfrac{e^{-s}(1+e^{-2s})}{s^2-s}$

(c) $F(s) = \dfrac{1}{e^{2s}(s^3+2s^2+6s)}$

3. Is it possible for $F(s) = \dfrac{s(s^2+3s-6)}{4s^3-3s+10}$ to be the Laplace transform for a piecewise continuous function of exponential order?

Answers

1. (a) $e^{-3(s+2)}\left[\dfrac{\cos 12(s+2) - 4\sin 12}{s^2+4s+20}\right]$

(b) $\dfrac{2}{s^2} - \dfrac{5}{s} + e^{-4s}\left(\dfrac{2}{s^3}+\dfrac{6}{s^2}+\dfrac{13}{s}\right) - e^{-8s}\left(\dfrac{2}{s^3}+\dfrac{16}{s^2}+\dfrac{63}{s}\right)$

(c) $\dfrac{1}{1-e^{-2s}}\left[\left(\dfrac{2}{s^3}-\dfrac{2}{s^2}+\dfrac{3}{s}\right) - e^{-2s}\left(\dfrac{2}{s^3}+\dfrac{2}{s^2}+\dfrac{3}{s}\right)\right]$

2. (a) $3 - (2+4t)e^{-t}$

(b) $(e^{t-1}-1)h(t-1) + (e^{t-3}-1)h(t-3)$

(c) $\dfrac{1}{6}\left\{1 - e^{2-t}\left[\dfrac{1}{\sqrt{5}}\sin\sqrt{5}(t-2) + \cos\sqrt{5}(t-2)\right]\right\} h(t-2)$

3. No