MATH 2132 Tutorial 12

In questions 1–5, use Laplace transforms to solve the differential equation subjected to whatever boundary and/or initial conditions are present.

1. $y'' + 3y' + 7y = 3\sin 2t, \quad y(0) = 1, \quad y'(0) = 2$

2. $y'' + 4y' + 4y = te^{-2t}, \quad y(0) = -1, \quad y'(0) = 0$

3. $y'' + 20y = t^2 - e^{-t}, \quad y(0) = 0, \quad y'(0) = 2$

4. $y'' - 2y' + y = \cos 2t, \quad y(0) = 1, \quad y(\pi) = 1$

5. $y'' + 2y' + 7y = e^t \sin t$

6. A 200 gram mass is at rest on the end of a spring with constant 10 Newtons per metre. At $t = 0$, a force $f(t) = 4\sin 10t$ begins to act on the mass. During its subsequent motion damping equal to 1.5 times the velocity also acts on the mass. Find the displacement of the mass as a function of time.

7. A 100 gram mass is suspended from a spring with constant 40 Newtons per metre. At $t = 0$, the mass is lifted 10 centimetres above its equilibrium position and given velocity 2 metres per second downward. During its subsequent motion damping equal to the velocity also acts on the mass. In addition, after 4 seconds, a constant force of 100 Newtons acts vertically upward on the mass. Find the displacement of the mass as a function of time.

Answers:

1. $e^{-3t/2}\left(\frac{7}{5}\cos\frac{\sqrt{19}t}{2} + \frac{37}{5\sqrt{19}}\sin\frac{\sqrt{19}t}{2}\right) + \frac{1}{5}\sin 2t - \frac{2}{5}\cos 2t$

2. $e^{-2t}\left(-1 - 2t + \frac{t^3}{6}\right)$

3. $\frac{t^2}{20} - \frac{1}{200} - \frac{1}{21}e^{-t} + \frac{221}{4200}\cos 2\sqrt{5}t + \frac{41}{42\sqrt{5}}\sin 2\sqrt{5}t$

4. $\frac{28}{25}e^t + \frac{28(1-e^\pi)}{25\pi e^\pi}te^t - \frac{3}{25}\cos 2t - \frac{4}{25}\sin 2t$

5. $\frac{e^t}{97}(9\sin t - 4\cos t) + e^{-t}(C_1\cos\sqrt{6}t + C_2\sin\sqrt{6}t)$

6. $-\frac{1}{65}(12\cos 10t + 8\sin 10t) + \frac{e^{-15t/4}}{65}\left(12\cos\frac{5\sqrt{23}t}{4} + \frac{100}{\sqrt{23}}\sin\frac{5\sqrt{23}t}{4}\right)$ m

7. $\frac{e^{-5t}}{10}\left(\cos 5\sqrt{15}t - \frac{\sqrt{15}}{5}\sin 5\sqrt{15}t\right)$
$+\frac{5}{2}\left\{1 - e^{5(4-t)}\left[\cos 5\sqrt{15}(t-4) + \frac{1}{\sqrt{15}}\sin 5\sqrt{15}(t-4)\right]\right\}h(t-4)$ m