

## MATH 2132 Tutorial 6

1. Use (a) differentiation and (b) binomial expansion to find the Taylor series about  $x = 1$  for  $1/(2x - 3)^3$ . What is the interval of convergence of the series?
2. Find the Taylor series for  $(15 - 2x)^{3/2}$  about  $x = -2$ . Express your answer in sigma notation, simplified as much as possible. What is the open interval of convergence?
3. Use series to evaluate

$$\lim_{x \rightarrow 0} \frac{\sqrt{3 + 4x^3} - \sqrt{3}}{x^3}.$$

4. Find a four-decimal approximation for

$$\int_0^1 \cos(x^2/4) dx.$$

Verify that your answer has the required accuracy.

5. Find a polynomial that approximates the antiderivative

$$\int (1 + x^4)^{-1/4} dx$$

on the interval  $0 < x < 1/2$  with error less than  $10^{-6}$ . Justify your answer.

6. Find a maximum possible error in using

$$\ln 7 + \frac{2}{7}(x - 1) - \frac{2^2}{2 \cdot 7^2}(x - 1)^2 + \frac{2^3}{3 \cdot 7^3}(x - 1)^3$$

to approximate the function  $\ln(2x + 5)$  on the interval  $-0.1 \leq x \leq 0$ . Justify your conclusions.

### Answers:

1.  $\sum_{n=0}^{\infty} -2^{n-1}(n+2)(n+1)(x-1)^n$ ,  $\frac{1}{2} < x < \frac{3}{2}$
2.  $\sqrt{19} \left\{ 19 - 3(x+2) + \sum_{n=2}^{\infty} \frac{3(2n-4)!}{2^{n-2}n!(n-2)!19^{n-1}}(x+2)^n \right\}$ ,  $-\frac{23}{2} < x < \frac{15}{2}$
3.  $2/\sqrt{3}$
4. 0.9938
5.  $x - \frac{x^5}{20} + \frac{5x^9}{288} - \frac{15x^{13}}{1664}$
6. 0.011