

MATH 2132 Tutorial 8

- Find a general solution of the differential equation $xy'' = x^3 - y'$. Explain why your solution is a general solution.
- Solve the initial-value problem

$$y'' = 4yy', \quad y(0) = 1, \quad y'(0) = 0.$$

Find general solutions for the differential equations in questions 3–5.

- $3y''' + 8y'' + 19y' + 10y = 0$
- $6y'''' + y'' - y = 0$
- $y'' + 3y' + ay = 0$, where $a > 3$ is a constant
- The roots of the auxiliary equation $\phi(m) = 0$ associated with the differential equation $\phi(D)y = 0$ are

$$2 \pm \sqrt{3}i, \quad 2 \pm \sqrt{3}i, \quad \pm 4, \quad \pm 4, \quad \pm 4, \quad 2, \quad -1 \pm \sqrt{6}.$$

What is a general solution of the differential equation?

- (a) Prove that $y(x) = C_1x^2 + C_2x^3 + (x^4/2)\ln x - 3x^4/4$ is a general solution of the differential equation

$$x^2y'' - 4xy' + 6y = x^4 \ln x.$$

- Find two linearly independent solutions of the associated homogeneous differential equation.
- Find a particular solution of the nonhomogeneous differential equation.

Answers:

- $y(x) = x^4/16 + C \ln|x| + D$
- $y(x) = 1$
- $y(x) = C_1e^{-2x/3} + e^{-x}(C_2 \cos 2x + C_3 \sin 2x)$
- $y(x) = C_1 \cos(x/\sqrt{2}) + C_2 \sin(x/\sqrt{2}) + C_3e^{x/\sqrt{3}} + C_4e^{-x/\sqrt{3}}$
- $y(x) = e^{-3x/2} \left[C_1 \cos \frac{\sqrt{4a-9}x}{2} + C_2 \sin \frac{\sqrt{4a-9}x}{2} \right]$
- $y(x) = e^{2x}[(C_1 + C_2x) \cos \sqrt{3}x + (C_3 + C_4x) \sin \sqrt{3}x] + (C_5 + C_6x + C_7x^2)e^{4x} + (C_8 + C_9x + C_{10}x^2)e^{-4x} + C_{11}e^{2x} + C_{12}e^{(-1+\sqrt{6})x} + C_{13}e^{-(1+\sqrt{6})x}$
- (b) x^2, x^3 (c) $(x^4/2)\ln x - 3x^4/4$