Assignment 4/5 Statistics 5.301 Due: Nov. 29

1. Two decision rules are given here. Assume they apply to a normally distributed quality characteristic, the control chart has three-sigma control limits, and the sample size is n=5

   Rule 1: If *one* or *more* of the next seven samples yield values of the sample average that fall outside the control limits, conclude that the process is out of control.

   Rule 2: If *all* of the next seven sample averages fall on the same side of the center line, conclude that the process is out of control.

   What is the type I error probability for each of these rules?

2. The net weight (in oz) of a dry bleach product is to be monitored by $\bar{x}$ and R control charts using a sample size of n=5. Data for 20 preliminary samples are as follows.

Sample Number	x_1	x_2	x_3	x_4	x_5
1	15.8	16.3	16.2	16.1	16.6
2	16.3	15.9	15.9	16.2	16.4
3	16.1	16.2	16.5	16.4	16.3
4	16.3	16.2	15.9	16.4	16.2
5	16.1	16.1	16.4	16.5	16.0
6	16.1	15.8	16.7	16.6	16.4
7	16.1	16.3	16.5	16.1	16.5
8	16.2	16.1	16.2	16.1	16.3
9	16.3	16.2	16.4	16.3	16.5
10	16.6	16.3	16.4	16.1	16.5
11	16.2	16.4	15.9	16.3	16.4
12	15.9	16.6	16.7	16.2	16.5
13	16.4	16.1	16.6	16.4	16.1
14	16.5	16.3	16.2	16.3	16.4
15	16.4	16.1	16.3	16.2	16.2
16	16.0	16.2	16.3	16.3	16.2
17	16.4	16.2	16.4	16.3	16.2
18	16.0	16.2	16.4	16.5	16.1
19	16.4	16.0	16.3	16.4	16.4
20	16.4	16.4	16.4	16.0	15.8

(a) Set up $\bar{x}$ and R control charts using these data. Does the process exhibit statistical control?

(b) Estimate the process mean and standard deviation.

(c) Does fill weight seem to follow a normal distribution?

(d) If the specifications are at 16.2 $\pm$ 0.5, what conclusions would you draw about process capability?

(e) What fraction of containers produced by this process is likely to be below the lower specification limit of 15.7 oz.?

3. Samples of $n = 8$ items each are taken from a manufacturing process at regular intervals. A quality characteristic is measured, and $\bar{x}$ and R values are calculated for each sample. After 50 samples we have

   $\sum_{i=1}^{50} \bar{x}_i$ = 2000 and $\sum_{i=1}^{50} R_i$ = 250

   Assume that the quality characteristic is normally distributed.

   (a) Compute control limits for the $\bar{x}$ and R control charts.

   (b) All points on both control charts fall between the control limits computed in part (a). What are the natural tolerance limits of the process?

   (c) If the specifications limits are 41 $\pm$ 5.0, what are your conclusions regarding the ability of the process to produce items within these specifications?

   (d) Assume that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process producing?

   (e) Make suggestions as to how the process performance could be improved.

4. Samples of n = 6 items each are taken from a manufacturing process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and S values are calculated for each sample. After 50 subgroups have been analyzed, we have

   $\sum_{i=1}^{50} \bar{x}_i$ = 1000 and $\sum_{i=1}^{50} S_i$ = 75

   (a) Compute control limits for the $\bar{x}$ and R control charts.

   (b) Assume that all points on both control charts plot within the control limits. What are the natural tolerance limits of the process?

(c) If the specifications limits are 19 ± 4.0, what are your conclusions regarding the ability of the process to produce items confirming to specifications?

(d) Assuming that if an item exceeds the upper specification limit it can be reworked, and if it is below the lower specification limit it must be scrapped, what percent scrap and rework is the process now producing?

(e) If the process were centered at μ = 19.0, what would be the effect on percent scrap and rework?

5. Consider the $\bar{x}$ and R charts you established in Exercise 5-1 using n = 5(Do the Exercise first, information given on a separate sheet) .

   (a) Suppose that you wished to continue charting this quality characteristic using $\bar{x}$ and R charts based on a sample size of n = 3. What limits would be used on the $\bar{x}$ and R charts?

   (b)What would be the impact of the decision you made in part (a) on the ability of the $\bar{x}$ chart to detect a 2σ shift in the mean?

   (c) Suppose you wished to continue charting this quality characteristic using $\bar{x}$ and R charts based on a sample size of n =8. What limits would be used on the $\bar{x}$ and R charts?

   (d) What is the impact of using n = 8 on the ability of the $\bar{x}$ chart to detect a 2σ shift in the mean?

6. Control charts for $\bar{x}$ and R are maintained for an important quality characteristic. the sample size is n = 7; $\bar{x}$ and R are computed for each sample. After 35 samples, we have found that

   $\sum_{i=1}^{35} \bar{x}_i$ = 7805 and $\sum_{i=1}^{35} R_i$ = 1200

   (a) Set up $\bar{x}$ and R charts using these data.

   (b) Assume that both charts exhibit control, estimate the process mean and standard deviation.

   (c) If the quality characteristic is normally distributed and if the specifications are 220 ± 35, can the process meet the specifications? Estimate the fraction nonconforming.

   (d) Assuming the variance to remain constant, state where the process mean should be located to minimize the fraction nonconforming. What would be the value of the fraction nonconforming under these conditions?

7. Samples of size n = 5 are taken from a manufacturing process every hour. A quality characteristic is measured and $\bar{x}$ and R are computed for each sample. After 25 samples have been analyzed, We have

   $\sum_{i=1}^{25} \bar{x}_i$ = 662.50 and $\sum_{i=1}^{25} R_i$ = 9.00

   The quality characteristic is normally distributed.

   (a) Find the control limits for the $\bar{x}$ and R charts.

   (b) Assume that both charts exhibit control. If the specifications are 26.40 $\pm$ 0.50, estimate the fraction nonconforming.

   (c) If the mean of the process were 26.40, what fraction nonconforming would result?

8. Samples of size n = 5 are collected from a process every half hour. After 50 samples have been collected, we calculate $\bar{\bar{x}}$ = 20.0 and $\bar{S}$ = 1.5. Assume that both charts exhibit control and that the quality characteristic is normally distributed.

   (a) Estimate the process standard deviation.

   (b) Find the control limits on the $\bar{x}$ and S charts.

   (c) If the process mean shifts to 22, what is the probability of concluding that the process is still in control?

9. An $\bar{x}$ is used to control the mean of a normally distributed quality characteristic. It is known that σ = 6.0 and n = 4. The center line = 200, UCL = 209, and LCL = 191. If the process mean shifts to 188, find the probability that this shift is detected on the first subsequent sample.

10. A parameter of a ap part being produced on a lathe has specifications 100 $\pm$ 10. Control chart analysis indicates that the process is in control with $\bar{\bar{x}}$ = 104 and $\bar{R}$ = 9.30. The control charts use samples of size n = 5. If we assume that the characteristic is normally distributed, can the mean be located (by adjusting the tool position) so that all output meets specifications? What is the present capability of the process?

11. A process is to be monitored with standard values μ = 10 and σ = 2.5. The sample size is three.

    (a) Find the center line and control limits for the $\bar{x}$ chart.

(b) Find the center line and control limits for the R chart.

(c) Find the center line and control limits for the S chart.

12. Control charts for $\bar{x}$ and R are in use with the following parameters:

$\bar{\mathbf{x}}$ Chart	**R Chart**
UCL $= 363.0$	**UCL** $= 16.18$
Center line $= 360.0$	**Center line** $= 8.91$
LCL $= 357.0$	**LCL** $= 1.64$

The sample size is n = 9. Both charts exhibit control. The quality characteristic is normally distributed.

(a) What is the α-risk associated with the $\bar{x}$ chart?

(b) Specifications on this quality characteristic are 358 $\pm$ 6. what are your conclusions regarding the ability of the process to produce items within specifications?

(c)Suppose the mean shifts to 357. What is the probability that the shift will not be detected on the first sample following the shift?

(d) What would be the appropriate control limits for the $\bar{x}$ chart if the type I error probability were to be 0.01?

13. A high-voltage power supply should have a nominal output voltage of 350V. A sample of four units is selected each day and tested for process-control purposes. The data shown give the difference between the observed reading on each unit and the nominal voltage times ten; that is,

$x_i =$ (observed voltage on unit i - 350)10

Sample Number	x_1	x_2	x_3	x_4
1	6	9	10	15
2	10	4	6	11
3	7	8	10	5
4	8	9	6	13
5	9	10	7	13
6	12	11	10	10
7	16	10	8	9
8	7	5	10	4
9	9	7	8	12
10	15	16	10	13
11	8	12	14	16
12	6	13	9	11
13	16	9	13	15
14	7	13	10	12
15	11	7	10	16
16	15	10	11	14
17	9	8	12	10
18	15	7	10	11
19	8	6	9	12
20	14	15	12	16

(a) Set up $\bar{x}$ and R charts on this process. Is the process in statistical control?

(b) If specifications are at $350V \pm 5V$, what can you say about process capability?

c) Is there evidence to support the claim that voltage is normally distributed?