PRACTICE QUESTIONS FOR TEST 1

The questions below are a *little* harder than the actual test questions. This is deliberate. If you practice with the hard questions, then you can certainly do the easier ones. Needless to say, you should also solve the assigned homework questions. In any event, do not expect the actual test questions to be copies of the practice questions.

The solutions to the practice questions will not be posted. If you wish to cross-check your solutions, then you should see me.

Consider the formula

$$\sum_{i=1}^{n} (i+3)^2 = \frac{n(2n^2 + 21n + 73)}{6}.$$

Q1. Prove this formula using mathematical induction.

Q2. Prove the same formula using the Bernoulli formulae.

Q3. Find all solutions to the equation

$$(1+i\sqrt{3})z^4 = 2.$$

Q4. Find a formula for $\sin(2A+B)$ in terms of $\sin A$, $\sin B$, $\cos A$, $\cos B$. Hint: consider $(e^{iA})^2 e^{iB}$.

Q5. Write the complex number

$$\left[\frac{\overline{4-i}}{\left(3+2\,i\right)^2}\right]$$

in Cartesian form.

Q6. Find the remainder when the polynomial $x^3 + 3x^2 - 5x + 1$ is divided by 2x - i.

Q7. Use mathematical induction to show that

$$1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^{2n}} = \frac{3}{2} \left[1 - \left(\frac{1}{3}\right)^{2n+1} \right].$$