MATH 1210: PRACTICE PROBLEMS FOR TEST 3

Q1. Let $A = \begin{bmatrix} 4 & -7 & 15 \\ -3 & 19 & 1 \\ 8 & 13 & -6 \end{bmatrix}$. Find $\det(A)$ in six different ways: i.e., by expanding along row 1, 2 or 3, or column 1, 2 or 3. Of course all of your answers must coincide.

Q2. Find the value of a such that the vectors

$$\mathbf{u}_1 = (3, a-1, -4), \quad \mathbf{u}_2 = (1, -a, 1), \quad \mathbf{u}_3 = (-5, 2, 3)$$

are linearly dependent. Now assuming this value for a, find explicit constants c_1, c_2, c_3 such that $c_1\,\mathbf{u}_1 + c_2\,\mathbf{u}_2 + c_3\,\mathbf{u}_3 = 0$.

Q3. Solve the system of equations

$$\begin{bmatrix} 3 & -2 & 5 \\ 6 & 1 & -8 \\ 4 & -2 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 26 \\ 60 \\ 33 \end{bmatrix}$$

by Cramer's rule. (You should always verify your answer by direct substitution.)

Q4. For the matrix $\begin{bmatrix} 9 & -5 & 1 & 7 \\ -8 & 2 & 4 & -3 \\ 1 & -4 & 8 & 11 \\ 15 & -6 & 10 & -1 \end{bmatrix}$, find the cofactors C_{12}, C_{43} as well as the minors M_{32}, M_{41}.

Q5. Let $C = \begin{bmatrix} 1 & x & 2 \\ x & 2 & 1 \\ 1 & 2 & x \end{bmatrix}$. Find all the values of x such that $\operatorname{rank}(C) < 3$. (There are three such values.)

Q6 (continuation of Q5). Let

$$\mathbf{u}_1 = (1, x, 1), \quad \mathbf{u}_2 = (x, 2, 2), \quad \mathbf{u}_3 = (2, 1, x).$$

(These are the columns of C.) For each of the values of x found in Q5, express $\mathbf{u}_1$ as a linear combination of $\mathbf{u}_2, \mathbf{u}_3$.

Q7. Let A be the matrix in Q1. Find $\det(-3\,A)$ and $\det(i\,A)$. Also find $\operatorname{adj}(A)$, and hence A^{-1}.

[I will not post the answers, although you are always welcome to come to my office for help. Most of the problems are such that if you get the correct answer, you will be able to verify it.]