MATH 1210: PRACTICE PROBLEMS FOR TEST 4

Q1. Find all the eigenpairs for each of the following matrices:

$$\begin{bmatrix} -2 & 3 \\ 5 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 9 & -1 \end{bmatrix}, \quad \begin{bmatrix} -5 & -14 & 14 \\ 6 & 18 & -17 \\ 6 & 16 & -15 \end{bmatrix}.$$

Q2. Find all the eigenvalues for the matrix $\begin{bmatrix} 7 & 40 & -45 \\ 3 & 27 & -28 \\ 4 & 30 & -32 \end{bmatrix}$.

Q3. Find all the eigenpairs of the symmetric matrix $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, and verify that the eigenvectors corresponding to different eigenvalues are orthogonal.

Q4. Same question as Q3 for the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & -2 \end{bmatrix}$. (This might be a little harder that Q3.)

Q5. Let $T : \mathbf{R}^3 \longrightarrow \mathbf{R}^2$ be a linear transformation such that

$$T(0,0,1) = (2,-4), \quad T(1,0,1) = (5,-3), \quad T(1,1,1) = (7,8).$$

Find the matrix of T. Moreover find a vector $\mathbf{v}$ such that $T(\mathbf{v}) = 0$.

Q6. Let $T : \mathbf{R}^3 \longrightarrow \mathbf{R}^3$ be a linear transformation such that

$$T(0,1,1) = (2,4,1), \quad T(1,0,1) = (3,7,-1), \quad T(1,0,0) = (1,-1,0).$$

Find the matrix of T. Moreover find a vector $\mathbf{v}$ such that $T(\mathbf{v}) = (2,1,2)$.

Q7. There are two cable providers in Winnipeg: Roger's and MTS. Assume that every month 31% of Roger's customers switch to MTS, while 19% of MTS customers switch to Roger's. In January, there are $10,000$ Roger's customers and 15000 MTS customers.

(a) Find the number of customers of each company in February and March.

(b) In the long run, how many customers belong to MTS and how many to Roger's ?

Assume that the total number of customers remains constant throughout.