11 December 2006 (afternoon) FINAL EXAMINATION

PAPER NO.: 200 PAGE NO.: 1 of 10 4 here

DEPARTMENT & COURSE NO: Mathematics - MATH 1210 Time: 2 hours

EXAMINATION: MATH 1210 - Techniques of Classical & Linear Algebra EXAMINER: T. G. Berry

VALUES

Instructions:

1. One hand-written single-sided reference page (8.5" by 11") is permitted. This information page must contain your name and student I.D. number.
2. No other aids are permitted.
3. **Attempt all problems. Show all your work.**

[11] 1. Use the Principle of Mathematical Induction to show that , for n any positive integer

$$\sum_{\ell=0}^{2n-1}(3\ell+1)=n(6n-1)\ .$$

[9] 2. Consider the complex numbers

$$z_1=\sqrt{2}(1+i)\ ,\quad z_2=3i\ \text{ and }\ z_3=\frac{1}{4}\left(-1+i\sqrt{3}\right)\ .$$

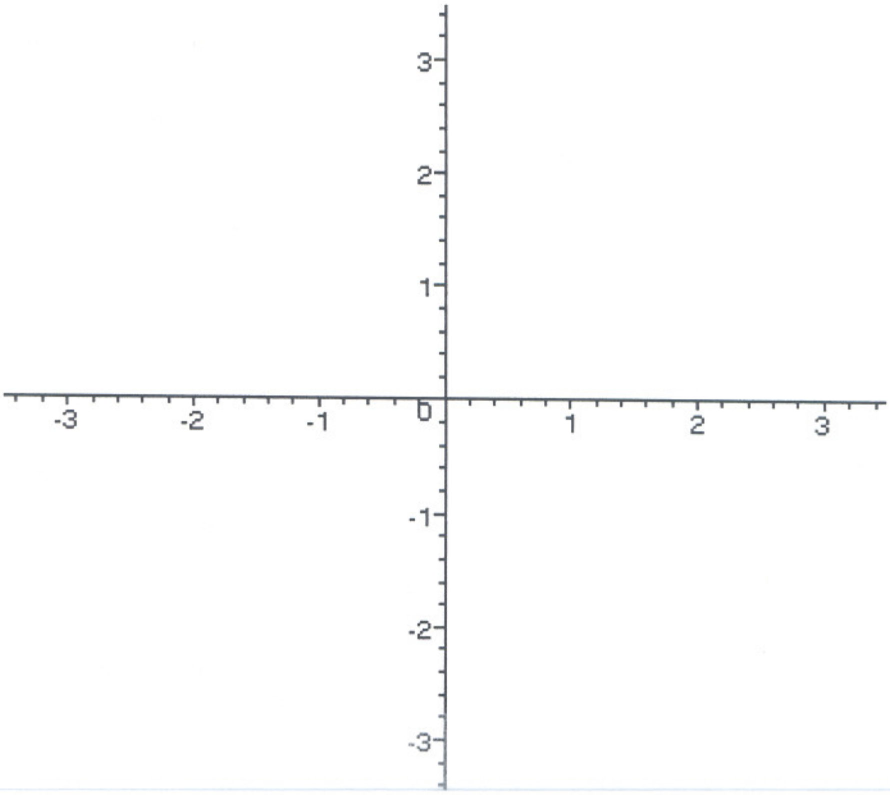
(a) Express each of these numbers in exponential form.

(b) Evaluate $\dfrac{z_1^{\,3}z_3}{z_2}$, expressing your answer in exponential form and simplifying the result as far as possible.

[10] 3. Consider the complex polynomial equation $z^4=16e^{i(3\pi/4)}$.

(a) Find the modulus and the principal value of the argument of each of the roots of this equation.

(b) Plot the roots determined in part (a) on the following diagram.



[9] 4. Consider the polynomial $P(x) = 2x^3 + x^2 + 5x - 3$.

(a) List all possible rational roots of $P(x) = 0$, as indicated by the Rational Roots Theorem.

(b) What is the maximum number of negative real zeros that $P(x)$ can possess, according to Descartes' Rule of Signs?

(c) Clearly Descartes' Rule of Signs indicates that $P(x)$ can have at most one positive real zero. Find it.

(d) Use the above information to find the remaining zeros of $P(x)$.

[10] 5. Given $A=\begin{pmatrix}2 & 1\\ -1 & 1\end{pmatrix}$, $B=\begin{pmatrix}1 & 3\\ -1 & 0\\ 2 & 1\end{pmatrix}$ and $C=\begin{pmatrix}1 & 1 & -1\\ 1 & 2 & 0\end{pmatrix}$, find

(a) $A\left(B-C^{T}\right)^{T}$

(b) $(CB)^{-1}$

[9] 6. Consider the matrix $\begin{pmatrix}1 & 0 & 0 & 0 & 4 & -2\\ 0 & 3 & 3 & -6 & 0 & 3\\ 0 & 0 & -1 & -4 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0 & 2 & 6\end{pmatrix}$.

(a) Reduce this matrix to row-echelon form (REF), indicating clearly all elementary row operations (ERO) used to do so. (You may combine some opeartions if you wish, as long as you indicate which operations you use.)

(b) Assuming that this matrix is the **augmented matrix** of **a non-homogeneous linear system** of equations of the form $Ax=b$, fill in the following table:

# of variables in this system	
rank of the augmented matrix	
rank of the coefficient matrix	
# of "free" variables in this system	
# of solutions possessed by this system	

(c) Assuming that this matrix is the **coefficient matrix** of **a homogeneous linear system** of equations of the form $Bx=0$, fill in the following table:

# of variables in this system	
rank of the augmented matrix	
rank of the coefficient matrix	
# of "free" variables in this system	
# of solutions possessed by this system	

[7] 7. Is the set of vectors consisting of $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$ and $\vec{v}_4 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$

linearly dependent or linearly independent. Justify your answer.

[13] 8. Consider the matrix $F = \begin{pmatrix} -1 & 0 & 1 \\ 2 & 4 & -1 \\ 0 & 2 & 1 \end{pmatrix}$.

(a) Evaluate $\det F$ using a **cofactor expansion along the second column**, showing all the details of the calculation.

(b) Find F^{-1}. (You may use any method you wish, but must show all your work.)

[12] 9. Consider the linear transformation $y = Ax$ in which $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

and $y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$.

(a) Find all the eigenvalues of A.

(b) Find all eigenvectors corresponding to each of the eigenvalues of A.

THE END! HAVE A GOOD VACATION.

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