

Cover sheet

Instructions

No aids.

There are seven pages, one question on each of pages 2 to 7. You should attempt all questions.

Answer all questions on the examination paper. If you do not have enough room, use the back of the *previous* page.

Question Number 1

Value 10 out of 65

(a) Write down the statement,

‘the sum of the squares of the positive integers from 1 to $3n$ is $n(3n + 1)(6n + 1)/2$ ’
using sigma notation.

(b) Prove the statement of part (a) for all positive integers n .

Question Number 2

Value 10 out of 65

Determine whether the four vectors

$$\mathbf{u} = (1, 1, -2, 0), \quad \mathbf{v} = (2, 1, -3, 2),$$

$$\mathbf{w} = (1, 2, -1, 4), \quad \mathbf{x} = (5, 2, 1, 3),$$

are linearly independent or linearly dependent.

Question Number 3

Value 10 out of 65

Find all solutions of the equations

$$\begin{aligned} x + y + 2z &= 5, \\ 2x + 3y - z &= 2, \\ 5x + 9y - 10z &= 6. \end{aligned}$$

Question Number 4

Value 15 out of 65

A linear transformation T is defined by

$$\begin{aligned} x' &= x + 2y + 2z, \\ y' &= 2y + z, \\ z' &= -x + 2y + 2z. \end{aligned}$$

Find all eigenvalues and eigenvectors for T .

Question Number 5

Value 10 out of 65

(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & -1 & 5 \\ 5 & 3 & -1 \\ -1 & 5 & 3 \end{bmatrix}.$$

(b) Use A^{-1} to write out the general solution of the equations

$$\begin{aligned} 3x - y + 5z &= a, \\ 5x + 3y - z &= b, \\ -x + 5y + 3z &= c; \end{aligned}$$

give expressions for the scalars x , y , and z in terms of the scalars a , b , and c .

Question Number 6

Value 10 out of 65

Find and list in the form $a + bi$, where a and b are real numbers, all roots of the equation $z^5 + z^4 - 16z - 16 = 0$, given that $z = -1$ is one root.

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