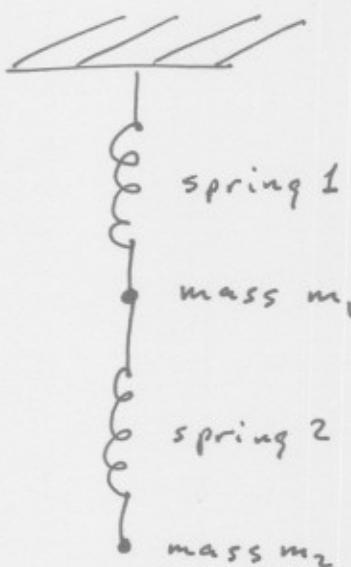
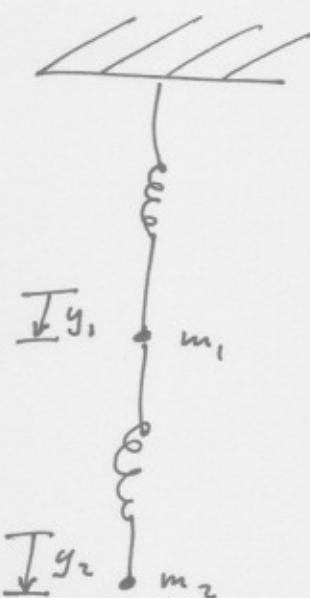


Eigen problem from Mechanics - Set-up

Consider a system of weightless and undamped springs:



Let the masses and springs be in static equilibrium and then the lower mass be drawn down by y_2 , lowering the upper mass by y_1 . The changes in spring lengths are y_1 and $y_2 - y_1$.



Let the constants associated with the springs be k_1 and k_2 so that the equations of motion of the masses are

$$m_1 \ddot{y}_1 \equiv m_1 \frac{d^2 y_1}{dt^2} = -k_1 y_1 + k_2(y_2 - y_1),$$

equal and opposite

$$m_2 \ddot{y}_2 \equiv m_2 \frac{d^2 y_2}{dt^2} = -k_2(y_2 - y_1).$$

The pair of scalar equations can be written

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} -\frac{k_1 + k_2}{m_1} & \frac{k_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \quad (1)$$

To solve this system, we let $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = e^{wt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$,

where w , x_1 , and x_2 are unknown scalars and t is time. Then $\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = w e^{wt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = w^2 e^{wt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

We substitute into both sides of (1), getting (2).