In MATH 3120 in the autumn of 2006, a correct answer to a problem was $3^n - 2\cdot 2^n + 1$.
Answers submitted by two of the best students in the class, obviously using slightly different methods, were:
$S_1 = \sum_{k=2}^{n}\left[\binom{n}{k}\sum_{j=1}^{k-1}\binom{k}{j}\right]$ and $S_2 = \sum_{k=1}^{n-1}\left[\binom{n}{k}\sum_{j=1}^{n-k}\binom{n-k}{j}\right]$.
Given that the symbol $\binom{n}{j}$ is defined to abbreviate $\dfrac{n!}{j!(n-j)!}$, where $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1 (n \geq 1)$ and $0! = 1$, and that both $\sum_{j=0}^{n}\binom{n}{j} = 2^n$, and $\sum_{j=0}^{n}\binom{n}{j}2^j = \sum_{j=0}^{n}\binom{n}{j}2^{n-j} = 3^n$, facts you will learn elsewhere, verify in complete detail that the two answers are both correct, i.e., that $S_1 = 3^n - 2\cdot 2^n + 1 = S_2$. The symbols $\binom{n}{j}$ are best left intact except in places where their numerical values are required, certainly always when they appear in a summation.

Hint. From $\sum_{j=0}^{n}\binom{n}{j} = 2^n$, $2^k = \binom{k}{0} + \sum_{j=1}^{k-1}\binom{k}{j} + \binom{k}{k} = 2 + \sum_{j=1}^{k-1}\binom{k}{j}$ so that
$S_1 = \sum_{k=2}^{n}\left[\binom{n}{k}(2^k - 2)\right] = \sum_{k=2}^{n}\binom{n}{k}2^k - 2\sum_{k=2}^{n}\binom{n}{k}$.

Rest of solution. From $\sum_{j=0}^{n}\binom{n}{j}2^j = 3^n$, $3^n = \binom{n}{0}2^0 + \binom{n}{1}2 + \sum_{k=2}^{n}\binom{n}{k}2^k = 1 + 2n + \sum_{k=2}^{n}\binom{n}{k}2^k$,
and from $\sum_{j=0}^{n}\binom{n}{j} = 2^n$, $2^n = \binom{n}{0} + \binom{n}{1} + \sum_{k=2}^{n}\binom{n}{k} = 1 + n + \sum_{k=2}^{n}\binom{n}{k}$, so that
$S_1 = 3^n - 2n - 1 - 2(2^n - n - 1) = 3^n - 2\cdot 2^n + 1$ as required.
From $\sum_{j=0}^{n}\binom{n}{j} = 2^n$, $2^{n-k} = \binom{n-k}{0} + \sum_{j=1}^{n-k}\binom{n-k}{j} = 1 + \sum_{j=1}^{n-k}\binom{n-k}{j}$ so that
$S_2 = \sum_{k=1}^{n-1}\left[\binom{n}{k}(2^{n-k} - 1)\right] = \sum_{k=1}^{n-1}\binom{n}{k}2^{n-k} - \sum_{k=1}^{n-1}\binom{n}{k}$.
From $\sum_{j=0}^{n}\binom{n}{j}2^{n-j} = 3^n$, $3^n = \binom{n}{0}2^n + \sum_{k=1}^{n-1}\binom{n}{k}2^{n-k} + \binom{n}{n}2^0 = 2^n + \sum_{k=1}^{n-1}\binom{n}{k}2^{n-k} + 1$,
and from $\sum_{j=0}^{n}\binom{n}{j} = 2^n$, $2^n = \binom{n}{0} + \sum_{k=1}^{n-1}\binom{n}{k} + \binom{n}{n} = 2 + \sum_{k=1}^{n-1}\binom{n}{k}$, so that
$S_2 = 3^n - 2^n - 1 - (2^n - 2) = 3^n - 2\cdot 2^n + 1$ as required.

Further comment. The facts $\sum_{j=0}^{n}\binom{n}{j} = 2^n$ and $\sum_{j=0}^{n}\binom{n}{j}2^j = \sum_{j=0}^{n}\binom{n}{j}2^{n-j} = 3^n$ are special cases for x and y equal to 1 and 2 of the famous binomial theorem, $(x+y)^n = \sum_{j=0}^{n}\binom{n}{j}x^j y^{n-j}$, which is fairly easily proved by induction using the fact that $\binom{n}{j} + \binom{n}{j-1} = \binom{n+1}{j}$, itself easily verified just using the above definition of $\binom{n}{j}$. Such a proof of the binomial theorem is available in binomialTheorem.pdf.