UNIVERSITY OF MANITOBA

DATE: June 12, 2007 | FINAL EXAMINATION
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COURSE: MATH 1210 | TIME: 2 hours
EXAMINATION: Classical and Linear Algebra | EXAMINER: M. Davidson

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1. The following are short answer questions.

[2] (a) What is the Cartesian form of $16e^{\frac{-\pi}{4}i}$?

[4] (b) What does Descartes' rule of signs imply about the polynomial $P(x) = 5x^4 - 4x^3 + 2x^2 + 7x - 13$?

[3] (c) Use the adjoint to find the inverse of the matrix $A = \begin{pmatrix} 3 & 5 \\ 1 & 4 \end{pmatrix}$.

[2] (d) Let T be the transformation from $\mathbb{R}^4$ to $\mathbb{R}^4$ defined by $T(\tilde{x}) = A\tilde{x}$ where $A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 7 & 1 \\ -1 & 7 & 6 & -2 \\ 3 & 1 & -2 & 1 \end{pmatrix}$ How many eigenvalues does T have? How many of them are real?

[2] (e) Write $3 - 3i$ in polar form.

[3] (f) Use the remainder theorem to find the remainder when the polynomial $P(x) = 3x^3 + 2x^2 - x + 3$ is divided by $x - 2i$.

[2] (g) Write the following in sigma notation (do not evaluate) : $1 - 3 + 5 - 7 + 9 - 11 + 13 - 15$

[4] (h) Are the vectors $\{(1, 1, 0), (2, 3, 4), (-1, 2, 6)\}$ linearly dependent or linearly independent. Justify your answer.

[3] (i) Given $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ evaluate the following :

$$\sum_{i=1}^{43}(i + 17)$$

[2] (j) Given $A = \begin{pmatrix} 0 & -2 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 & 5 \\ 1 & 2 & 1 \end{pmatrix}$ then $AB = \begin{pmatrix} -2 & -4 & -2 \\ 5 & 3 & 7 \end{pmatrix}$. What is $B^T A^T$?

[2] (k) Are the vectors $\{(1, 3), (2, -5), (6, 7)\}$ linearly dependent or linearly independent. Justify your answer.

[3] (l) If $z = 7 + 7i$, what is z^3? (hint: this may be easier using DeMoivre's theorem.)

[12] 2. Use mathematical induction to show that for all $n \geq 1$ that

$$1 + 3 + 5 + \ldots + (4n - 1) = (2n)^2 .$$

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[10] 3. Find all solutions to $w^3 = -4\sqrt{2} + 4\sqrt{2}i$. Give your answers in exponential form.

[10] 4. Find all roots of the polynomial $P(x) = x^3 - 5x^2 + 11x - 15$. (hint: Start by considering the rational roots)

[10] 5. Use Cramer's rule to find the solution to the system of equations:

$$\begin{array}{rcrcrl} 2x & + & y & + & 2z = & 1 \\ 3x & - & y & + & 4z = & -8 \\ 5x & + & 4y & + & 3z = & 11 \end{array}$$

[12] 6. (a) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 10 & 4 \\ 2 & 8 & 5 \end{pmatrix}$.

(b) Use the information from part a to find a solution to :

$$\begin{array}{rcrcrl} x_1 & + & 3x_2 & + & x_3 = & 1 \\ 3x_1 & + & 10x_2 & + & 4x_3 = & -1 \\ 2x_1 & + & 8x_2 & + & 5x_3 = & 3 \end{array}$$

[14] 7. Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(\tilde{x}) = A\tilde{x}$ where $A = \begin{pmatrix} -1 & 7 & -7 \\ 0 & 2 & -3 \\ 0 & -4 & 3 \end{pmatrix}$. Find all eigenvalues of T. Find all eigenvectors associated with each eigenvalue of T.