[20] 1. Consider the polynomial $P(x) = 3\,x^4 + 10\,x^3 + 24\,x^2 + 22\,x + 5$.

    (a) Apply Descartes' rules of signs to $P(x)$; what possible number of real positive roots does $P(x)$ have? Negative roots?

       Answer: 0 positive roots because of 0 sign changes.

       $P(-x) = 3\,x^4 - 10\,x^3 + 24\,x^2 - 22\,x + 5$.

       4 or 2 or 0 negative roots because of 4 sign changes in $P(-x)$.

    (b) Apply the bounds theorem to $P(x)$ (be specific about what it gives).

       Answer: Real roots are bounded in absolute value by ± 9.

    (c) What are the possible rational roots of $P(x)$? Take account of any limitations from part a and/or part b.

       Answer: $-1, -5, -1/3, -5/3$.

    (d) Find all roots of $P(x) = 3\,x^4 + 10\,x^3 + 24\,x^2 + 22\,x + 5$.

       Answer: $-1, -1/3, -1 \pm 2i$.

[12] 2. (a) Give the parametric equation of a line ℓ_1 that passes through the points $(4, 2, -1)$ and $(5, 0, 2)$.

$$\text{Answer: } \ell_1 : \mathbf{x} = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}.$$

    (b) Find, if it exists, the point of intersection of ℓ_1 (from above) and the line

$$\ell_2 : \mathbf{x} = \begin{bmatrix} 1 \\ -3 \\ 6 \end{bmatrix} + t \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}.$$

       Answer: $(6, -2, 5)$ corresponding to $s = 2$ and $t = 1$.

[8] 3. For each of the following pairs $\vec{u}, \vec{v}$, find $\vec{u} \cdot \vec{v}$. Is the pair of vectors orthogonal?

    (a) $\vec{u} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$.

       Answer: $\vec{u} \cdot \vec{v} = 5$. Not orthogonal.

    (b) $\vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$.

       Answer: $\vec{u} \cdot \vec{v} = 0$. Orthogonal.

    (c) $\vec{u} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$.

       Answer: $\vec{u} \cdot \vec{v} = 0$. Orthogonal.

    (d) $\vec{u} = \begin{bmatrix} 7 \\ -3 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ -3 \\ -6 \end{bmatrix}$.

       Answer: $\vec{u} \cdot \vec{v} = 18$. Not orthogonal.

[10] 4. Let $A = \begin{pmatrix} 3 & -4 \\ 2 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -1 & 5 \\ 3 & 0 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 3 & -1 \\ 2 & 0 \\ -3 & 5 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 3 & -5 \\ 1 & 0 & -1 \\ -4 & 2 & 5 \end{pmatrix}$, $E = \begin{pmatrix} -1 & 3 & 2 \end{pmatrix}$ and $F = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$.

Evaluate each expression below, if it is defined. If it is undefined, state that it is undefined and explain why.

(a) $BC + A^T$

Answer: $\begin{pmatrix} -2 & 23 \\ -13 & 34 \end{pmatrix}$

(b) $EF + D$

Answer: Undefined because EF and D have different dimensions.

(c) $(E + F^T)C$

Answer: $\begin{pmatrix} 25 & -9 \end{pmatrix}$.