Paper No. 302 MATH 1210

The University of Manitoba

Classical and Linear Algebra

Final Examination: Fall 2009.

December 14th, 2009.

Time: Two hours Total Marks: 100

Last Name (CAPITALS LETTERS ONLY): ______________________

Student Number: ______________________

First Name (CAPITAL LETTERS ONLY): ______________________

Signature: ______________________

(I acknowledge that cheating is an extremely serious offense.)

Place a check mark (✓) in the box corresponding to your instructor and section.

- ☐ T. Berry (A01)
- ☐ J. Chipalkatti (A02)
- ☐ Challenge for credit
- ☐ Deferred

Instructions:

Please ensure that your paper has a total of **11** pages (including this page). Read the questions thoroughly and carefully before attempting them.

You are **not allowed** to use any of the following: calculators, notes, books, dictionaries or electronic communication devices (e.g., cellular phones, pagers or blackberries). You may use the **left-hand pages** for rough work.

	Obtained	Maximum
Page 2		13
Page 3		12
Page 4		10
Page 5		13
Page 6		10
Page 7		12
Page 8		7
Page 9		7
Pages 10-11		16
Total		**100**

Q1. Prove the following statement using the Principle of Mathematical Induction: [7]

$$\sum_{r=1}^{n} (-1)^r = \frac{(-1)^n - 1}{2}.$$

Q2. Consider the polynomial $p(x) = 7\,x^{27} - 34\,x^8 + 15\,x^2 - x + 10$. [4+2]

(a) List all the possible rational roots of $p(x)$.

Answer: ____________________________

(b) Use Descartes' rule of signs to determine the maximum number of positive and negative (real) roots.

Answer: The polynomial $p(x)$ has at most ______ positive roots, and at most ______ negative roots.

Q3. Let $p(x) = x^{2009} - 3\,x^{1492} + 2$. Find the remainder when $p(x)$ is divided by $x + i$. [4]

Remainder = ____________

Q4. Let $z_1 = 1 + i$, and $z_2 = -1 + i\sqrt{3}$. [4+4]

(a) Express z_1 and z_2 in exponential form.

$z_1 =$ __________ $z_2 =$ __________

(b) Find the modulus and the principal value of the argument of $z = z_1^2\, z_2$.

$|z| =$ __________ p.v. $\arg(z) =$ __________

Q5. Consider the lines L_1, L_2 in $\mathbb{R}^3$ given by the following parametric equations: [5]

$$L_1 : (x, y, z) = (7, 1, 4) + s\,(1, 1, 3),$$

and

$$L_2 : (x, y, z) = (3, 2, 0) + t\,(2, -3, -2).$$

Find the point of intersection of L_1 and L_2.

Answer: The point of intersection is __________

Q6. Let θ denote the angle between the vectors $u = (3, 0, -4)$ and $v = (1, 1, -1)$. Find the value of $\sin\theta$.

[5]

Answer: $\sin\theta =$ __________

Q7. Let L denote the line in $\mathbb{R}^3$ given by the symmetric equation

$$\frac{x-1}{3} = \frac{y-2}{-2} = \frac{z-5}{4},$$

and let $\mathcal{P}$ denote the plane $x+y+z=3$. Find the point at which L intersects $\mathcal{P}$. [5]

Answer: The point of intersection is __________

Q8. Let $A = \begin{bmatrix} 1 & 0 \\ i & -i \end{bmatrix}$, Find the following matrices in simplified form. [4+4]

(Note that $i = \sqrt{-1}$.)

(a) $A^2 =$

(b) $A^{-1} =$

Q9. Find the value of k for which the following system is consistent: [5]

$$3x - 2y = k, \quad 2x + y = 6, \quad -x + 3y = 4.$$

Answer: $k =$ _______

Q10. Find the value of the constant a such that the vectors [5]

$$(-1, 2, a), \quad (2, 7, 4), \quad (3, 5, -2a)$$

are linearly dependent.

Answer: $a =$ _______

Q11. Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$. [3+3]

(a) $\det(A) =$ _______

(b) Find the element in the 2nd row and 3rd column of A^{-1}.

Answer: _________

Q12. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ -3 & 4 & 2 \end{bmatrix}$. Use the direct method (i.e., the row-reduction method) to find A^{-1}.

[6]

Q13. Consider the matrix $A = \begin{bmatrix} 5 & 0 & -3 \\ -4 & 1 & 3 \\ 8 & 0 & -4 \end{bmatrix}$. [3+4]

(a) Find the characteristic polynomial of A.

Answer: ____________

(b) It is given that $\lambda = 1$ is one of the eigenvalues of A. Find the remaining two eigenvalues.

Answer: ________________

Q14. Consider the linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ given by the formula $T(v_1, v_2, v_3) = (2\,v_1 - v_2 + v_3, v_1 - 4\,v_3)$. [2+1+4]

(a) Write down the matrix corresponding to T.

(b) Find the image of the vector $(1, -6, 3)$ under T.

Answer: $T(1, -6, 3) =$ __________

(c) Find a **nonzero** vector v in $\mathbb{R}^3$ such that $T(v) = 0$.

Answer: $v =$ __________

Q15. Consider the symmetric matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$. It is given that 3 and 0 are two of the eigenvalues of A.

[5+5+2+4]

(a) Find an eigenvector u corresponding to the eigenvalue 3.

Answer: $u =$ ______

(b) Find an eigenvector v corresponding to the eigenvalue 0.

Answer: $v =$ ______

(Q15 continued ...)

(c) Find the angle between u and v.

Answer: The angle is _______

(d) Find the remaining eigenvalue of A.

Answer: $\lambda =$ _______