Attempt all questions and show all your work. Due September 24, 2010.

1. Use Mathematical Induction to prove that for all $n \geq 1$,
$$n + (n+1) + (n+2) + \cdots + (2n) = \frac{3n(n+1)}{2}.$$

2. Use Mathematical Induction to prove that for all $n \geq 1$,
$$\sum_{i=1}^{n} (i+3)^2 = \frac{n(2n^2 + 21n + 73)}{6}$$

3. Use Mathematical Induction to prove that for all $n \geq 1$,
$$1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^{2n}} = \frac{3}{2}\left(1 - \left(\frac{1}{3}\right)^{2n+1}\right).$$

4. (a) Write the sum $1 + 3 + 5 + \cdots + (4n-1)$ using sigma notation.
   (b) Use Mathematical Induction to prove that for all $n \geq 1$, the above expression is equal to $(2n)^2$.

5. Use Mathematical Induction to prove that for all $n \geq 1$, $3^n > n^2$.

6. Consider the sequence of real numbers defined by the relations $x_1 = 1$ and $x_{n+1} = \sqrt{1 + 2x_n}$ for $n \geq 1$. Use the Principle of Mathematical Induction to show that $x_n < 4$ for all $n \geq 1$.

7. Let a and d be fixed real numbers. Prove using Mathematical Induction that for each $n \geq 1$,
$$a + (a+d) + (a+2d) + \cdots + (a + (n-1)d) = \frac{n}{2}(2a + (n-1)d).$$

8. (a) Express the sum $\displaystyle\sum_{k=1}^{m} (2+3k)^2$ in terms of three simpler sums in sigma notation by expanding. Do not calculate the value.
   (b) Find the value of the sum
$$\sum_{p=1}^{100} (2 - 10p + 3p^2).$$
   HINT: Make use of the formulas
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \text{and} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$
   (c) Rewrite the sum
$$\sum_{r=12}^{122} \frac{r-6}{r+9}$$
   using an index whose initial and terminal values are 1 and 111 (HINT: use a change of variables).