Mathematics 1210 Assignment 2 Fall 2010

Hand in as per the instructions of your section lecturer by Friday October 8. Follow instructions in the "Test and Tutorial Information" document on the course webpage. It is recommended (but not required) that you write only on one side and attach pages with a single staple in the top left corner, without plastic covers, paper clips or other fasteners.

1. Simplify $\dfrac{169}{5+12i} + \left(\overline{(1-2i)^3+4}\right)^2$ and express in Cartesian form.

2. Express in the forms required, with all arguments in your answers reduced to numbers in the interval $(-\pi, \pi]$.

   (a) $-6+\sqrt{108}i$ in polar and exponential form;

   (b) $\sqrt{18}\left(\cos\dfrac{19\pi}{4} + i\sin\dfrac{19\pi}{4}\right)$ in Cartesian and exponential form;

   (c) $10e^{-\frac{5\pi}{6}}$ in Cartesian and polar form.

3. $\cos n\theta$, $n \in \mathbb{Z}$, can always be expressed in terms of $\sin\theta$ and $\cos\theta$. For example, $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$. Use De Moivre's Theorem to obtain an expression of this type for $\cos 7\theta$.

4. Find all of the complex 6th roots of -64. Express your answers in Cartesian form.

5. Solve the equation $x^4 - 8x^2 + 36 = 0$ over the complex numbers.

6. (a) Use long division to find the quotient and remainder when $x^5 - 3x^4 + 2x^2 - x + 7$ is divided by $x - 3$. Express the result as an equation in the form (polynomial) = (polynomial)$\cdot$(quotient)+(remainder).

   (b) Use the Remainder Theorem to find the remainder when $f(x) = (1+i)x^4 + 3ix^3 + (1-i)x + 2$ is divided by $ix - 3$. (Do not perform long division!)

   (c) For which value of d is the polynomial $2x - 3$ a factor of the polynomial $g(x) = x^3 - 5x^2 + 2x - d$?

   (d) You are given that $(x-2)$ and $(x+1)$ are factors of the polynomial $f(x) = x^4 - 8x^3 + hx^2 + kx + 6$. What are the values of h and k?

7. You are given that $2+i$ is a zero of the polynomial $p(x) = x^4 - 4x^3 + 9x^2 - 16x + 20$. Write $p(x)$ as a product of linear factors. What are the roots of equation $p(x) = 0$?

8. In each case your response should refer by number to appropriate results in Section 2.2.1 as needed.

   (a) If a polynomial of degree n with real coefficients does not have n real zeros (counting multiplicity) then it must have an irreducible quadratic factor. Justify this statement.

   (b) If r is a zero of polynomial $f(x)$ of multiplicity 5 and a zero of polynomial $g(x)$ of multiplicity 7, must it also be a zero of polynomial $h(x) = f(x) + g(x)$? If so, can we determine it's multiplicity? If so what is it? If not, why not?