MATH 1210 Assignment 5 11R-T1

Attempt all questions and show all your work. Hand in as per the instructions of your section lecturer by **Fri, December 3, 2010**. Follow instructions in the "Test and Tutorial Information" document on the course webpage. It is recommended (but not required) that you write only on one side and attach pages with a single staple in the top left corner, without plastic covers, paper clips or other fasteners.

1. Use the direct method to find the inverse of

$$\begin{bmatrix} 2 & 1 & -1 \\ -13 & -12 & 13 \\ 10 & 10 & -11 \end{bmatrix}$$

2. Find the adjoint of

$$\begin{bmatrix} 6 & 10 & 5 \\ -3 & -5 & -3 \\ -7 & -11 & -5 \end{bmatrix}$$

3. Use the adjoint method to find the inverse of

$$\begin{bmatrix} 0 & 3 & 5 & 4 \\ 7 & 0 & 2 & 0 \\ 10 & 0 & 3 & 0 \\ 0 & 1 & 9 & 0 \end{bmatrix}$$

4. (a) Find the inverse of $\begin{bmatrix} 6 & 5 \\ 9 & 7 \end{bmatrix}$.

   (b) Use part (a) to solve the system

$$\begin{array}{rcrcr} 6x & + & 5y & = & 3 \\ 9x & + & 7y & = & -2. \end{array}$$

   (c) Let $a, b \in \mathbb{R}$. Use part (a) to solve the system

$$\begin{array}{rcrcr} 6x & + & 5y & = & a \\ 9x & + & 7y & = & b. \end{array}$$

   (d) Use part (a) to find the solution to the matrix equation

$$\begin{bmatrix} 6 & 5 \\ 9 & 7 \end{bmatrix} X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

5. Let $V = \{[5, -7, -1, 2], [1, 1, 2, 6], [0, 2, 4, 6]\}$.

   (a) Express $[4, -4, 5, 8]$ as a linear combination of the vectors in V.

   (b) Prove $[1, 0, 0, 0]$ cannot be expressed as a linear combination of the vectors in V.

6. Show that **every** 3-tuple (vector in $\mathbb{E}^3$) can be expressed as a linear combination of the vectors $[1, 2, 3], [4, 5, 0], [6, 0, 0]$.

7. (a) Show that $\{[-5, 6, -4, -6], [-10, 4, 7, 3], [9, 4, 3, 7]\}$ is linearly independent.
   (b) Show that $\{[1, 9, -4, -10], [-4, 3, -2, 1], [14, -35, 0, 14]\}$ is linearly dependent.
   (c) Show that $\{[2, -5, 0, 2], [-4, 10, 0, -4], [-6, -4, -3, 1]\}$ is linearly dependent.

8. Let $V = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a linearly independent set of vectors. Prove (using the definition of linearly independent) that if we remove the first vector in the set, the remaining set is still linearly independent. That is, prove that $V' = \{\mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is linearly independent.

9. Let $V = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be some set of vectors in $\mathbb{E}^m$. Assume that $\mathbf{w}_1$ and $\mathbf{w}_2$ are two vectors that can be written as linear combinations of the vectors in V. Prove then that the vector $\mathbf{w}_1 + \mathbf{w}_2$ can also be written as a linear combination of the vectors in V.

10. For which values of a can every vector in $\mathbb{E}^3$ be expressed as a linear combination of the following vectors:
$$[a, 1, 1], [1, 1, a], [1, a, 1]?$$