UNIVERSITY OF MANITOBA

DATE: Jan 11, 2011 — DEFERRED FINAL EXAM TITLE PAGE

DEPARTMENT & COURSE NO: MATH 1210 — TIME: 3 hours

EXAMINATION: Techniques of Classical and Linear Algebra — EXAMINER: Craigen/Borgersen

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NAME: (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

Please indicate your instructor and section by placing a check mark in the appropriate box below.

☐ A01 (MWF 9:30) R. Craigen

☐ A02 (MWF 1:30) R. Borgersen

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please clearly show all necessary work.**

No texts or notes are permitted. Calculators are NOT permitted. Cell phones, electronic translators, and other electronic devices are **not** permitted.

This exam has a title page and 12 pages, including 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

If you need more scrap paper, use the back of the question pages.

Question	Points	Score
1	6	
2	7	
3	8	
4	4	
5	14	
6	12	
7	8	
8	10	
9	8	
10	15	
11	8	
12	12	
13	10	
Total:	122	

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Short Answer

[3] 1. (a) Write the following expression using sigma notation with an index starting at 1:

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots+\frac{n}{n+1}.$$

[3] (b) Write out the first three and last three terms of the sum $\sum_{k=-2}^{12} k^3(k-2)$.

2. Answer, and give a reason for your answer, for each of the following:

[2] (a) Must a polynomial of degree 2010 have a real zero?

[2] (b) Must a polynomial of degree 2011 have a real zero?

[3] (c) Does the polynomial $1+x^2+x^4+x^6+\cdots+x^{100}$ have a real zero?

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3. Let $A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 5 \\ 3 & -4 & -1 \end{pmatrix}$. Compute each of the following or state why it cannot be done.

[2] (a) BA^T

[2] (b) $B^T A^2$

[2] (c) $(2A + 3I_2)B$

[2] (d) $B^3 - B^2$

[4] 4. The augmented matrix of a system of equations in variables x_1, x_2, x_3, x_4 has the Row-Echelon form given below. Write down its solution.

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 1 & 1 & -3 \end{array}\right).$$

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5. Give all possibilities in each case, and briefly justify each answer.

[4] (a) How many solutions can a system of n equations in n variables have if its coefficient matrix is nonsingular?

[6] (b) How many solutions can the system have if it is homogeneous and its coefficient matrix is singular?

[4] (c) How many solutions can the system have if it is nonhomogeneous and its coefficient matrix is singular?

Long Answer

[12] 6. Prove using mathematical induction that for every $n \geq 1$,

$$\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{pmatrix}.$$

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[8] 7. Find all rational zeros of the polynomial $f(x) = 2x^3 - 5x^2 - 4x + 3$. Show all essential steps in your derivation of the answer.

[10] 8. Express $z = \left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ in Cartesian form.

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[8] 9. Solve the system of equations

$$\begin{aligned} x \quad\quad\quad + 8z &= 1 \\ x + y + 6z &= 2 \\ -2y + 4z &= 3, \end{aligned}$$

by putting its augmented matrix into reduced row echelon form.

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[8] 10. (a) Find the adjoint of the matrix $A = \begin{pmatrix} 2 & 2 & 2 \\ 4 & 6 & 5 \\ 6 & 3 & 3 \end{pmatrix}$.

[2] (b) Given that $\det A = -6$, how do we know that A is invertible?

[2] (c) Find A^{-1}.

[3] (d) Use your answer to (b) above to solve the system of equations

$$\begin{array}{rcrcrcr} 2x & + & 2y & + & 2z & = & 0 \\ 4x & + & 6y & + & 5z & = & 2 \\ 6x & + & 3y & + & 3z & = & -6 \end{array}$$

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[8] 11. Determine if the following vectors are linearly dependent or independent:

$$[1, 1, 1, 1], [1, 2, 0, -2], [0, 1, 1, 3], [1, -1, 2, 0].$$

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[6] 12. (a) Find parametric equations for the line in $\mathbb{E}^3$ passing through the points $X(1, 2, -3)$ and $Y(2, 2, 0)$.

[6] (b) Find an equation, in standard form, for the plane passing through the points $A(1, 1, 0)$ and perpendicular to the line obtained in part (a), in $\mathbb{E}^3$.

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[10] 13. Find all the eigenpairs of the linear transformation on $\mathbb{E}^3$ defined by

$$T[x, y, z] = [y + z, -x + 2y + 3z, 3z].$$

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