[4] 1. Write the following sum using sigma notation with an index starting at 1:

$$n^2 - (n-1)^2 + (n-2)^2 - (n-3)^2 + \cdots + (-1)^{n-1}(1)^2.$$

[2] 2. For each $n \geq 1$, the n-th triangular number is $T_n = 1 + 2 + \cdots + n$. Prove using Mathematical Induction that, for each $n \geq 2$,

$$T_n + T_{n-1} = n^2.$$

No marks will be awarded for any other method.

[3] 3. (a) Write the following complex number in Cartesian form: $5\sqrt{2}e^{\frac{15\pi}{4}i}$

[4] (b) Write $9\sqrt{3} - 9i$ in polar form.

[7] (c) Find the cube roots of $-4 + 4i$. Express your answers in exponential form.

4. Consider the polynomial $f(x) = 6x^5 - 3x^4 - 5x^2 + x + 4$. Without factoring this polynomial or attempting to find any zeros for it:

[4] (a) Use the Rational Root Theorem to write down all possible rational zeros of $f(x)$.

[3] (b) Use Descartes' Rules of Signs to determine how many positive real zeros $f(x)$ may have and how many negative real zeros $f(x)$ may have.

[3] (c) Use the Bounds Theorem to determine how large the absolute value of a root of $f(x)$ may be.

[3] (d) Use your answers to (c) and (d) to improve your answer to (a) (that is, taking in account your answers for (c) and (d), list all possible rational zeros of $f(x)$).

[3] (e) If $a + bi$ is a complex root of $f(x)$, where $b \neq 0$, find another root, and write down an irreducible real quadratic factor of $f(x)$.

5. Let $P = (1, -1, 0)$, $\vec{u} = [1, 2, -3]$ and $\vec{v} = [2, 0, 2]$.

[3] (a) Find $\cos\theta$ where θ is the angle between $\vec{u}$ and $\vec{v}$.

[3] (b) Give an equation in vector form for the line passing through P and with direction vector $\vec{v}$.

6. Let $\vec{x} = [1, 0, -4]$, $\vec{y} = [a, 1, 3]$.

[3] (a) Find all values of a such that $\vec{x}$ and $\vec{y}$ are perpendicular.

[5] (b) Find all values of a such that $||\vec{x} + 2\vec{y}|| = 3$.