UNIVERSITY OF MANITOBA

FINAL EXAM

DATE: December 19, 2011

DEPARTMENT & COURSE NO: MATH 1210

TIME: 2 hours

EXAMINATION: Techniques of Classical and Linear Algebra

EXAMINER: various

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1. Answer the following questions by filling in the blanks. The answers are required in the simplified form. **Note that only final answers will be marked.**

[3] (a) The number of terms in the sum $\sum_{k=4}^{2011} a_k$ is equal to ____________ .

[4] (b) Polar form of the complex number $z = -1 + i$ is ____________ .

[3] (c) The Cartesian form of the complex number $z = 12e^{i\pi}$ is ____________ .

[4] (d) When the polynomial $x^4 - 4x^3 + 3x^2 + x + 3$ is divided by the polynomial $x + 1$, the remainder is equal to ____________ .

[3] (e) According to the Bounds Theorem,
all roots of the polynomial $5x^5 - 3x^2 + 4x + 1$ satisfy $|x| <$ ____________ .

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[3] (f) If $\|\vec{u}\| = 4$, $\|\vec{v}\| = 5$, $\vec{u} \cdot \vec{v} = -10$,
then the angle between $\vec{u}$ and $\vec{v}$ is equal to _______________ .

[4] (g) If $\vec{u} = [1, 2, -1]$ and $\vec{v} = [-1, 3, 5]$, then $\vec{u} \times \vec{v} =$ _______________ .

[3] (h) A homogeneous system consists of 5 linear equations in 7 unknowns. If its coefficient matrix has rank 3, how many linearly independent basic solutions does the system have? Answer: _______________ .

[4] (i) The eigenvalues of the matrix $\begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}$ are: _______________ .

[3] (j) Suppose A is a symmetric matrix with eigenvalues $\lambda = 3, -2$. Let $\vec{u}$ be an eigenvector of A corresponding to the eigenvalue $\lambda = 3$, $\vec{v}$ be an eigenvector of A corresponding to the eigenvalue $\lambda = -2$. Then $\vec{u} \cdot \vec{v} =$ _______________ .

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[12] **2.** Use mathematical induction to show that

$$\sum_{j=1}^{n} j(j+2) = \frac{n(n+1)(2n+7)}{6}$$

for every integer $n \geq 1$.

[10] **3.** Using Cramer's rule (no credit for any other method), solve the system

$$\begin{aligned} 2x + 5y &= -1, \\ x - 3y &= 4. \end{aligned}$$

[17] **4.** Using the adjoint method (no credit for any other method), find the inverse of the matrix $A = \begin{pmatrix} 2 & 0 & -1 \\ 0 & -2 & 3 \\ 0 & -1 & 2 \end{pmatrix}$.

[13] **5.** Suppose $\vec{u} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$, $\vec{w} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$. Express the vector $\begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$ as a linear combination of the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$.

[14] **6.** Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(\vec{x}) = A\vec{x}$, where $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 3 \end{pmatrix}$.
It is known that T has eigenvalues $\lambda = 1, 2$ (one of the eigenvalues has multiplicity 2). Find all eigenvectors associated with each eigenvalue of T.