## UNIVERSITY OF MANITOBA DEPARTMENT OF MATHEMATICS MATH 1210 Techniques of Classical and Linear Algebra SECOND TERM TEST November 9, 2011 5:30 – 6:30 PM

Please indicate your instructor and section by checking the appropriate box below:

A01	MWF $(9:30 - 10:20 \text{ AM}, \text{ EITC E3 } 270)$	G. Krause
A02	MWF (1:30 – 2:20 PM, St. Paul's College $305$ )	A. Prymak
A03	MWF (1:30 – 2:20 PM, EITC E2 155)	M. Despic

## **INSTRUCTIONS TO STUDENTS:**

Fill in all the information above.

This is a 1 hour exam.

No notes, books, cell phones, calculators or other computing devices are permitted.

Show your work clearly for full marks.

This test has a title page, 6 pages of questions, and 1 blank page for rough work. Please check that you have all pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 100.

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the **reverse** side of the page, but **clearly indicate** that your work is continued there.

Question:	1	2	3	4	5	6	Total
Points:	27	17	17	13	19	7	100
Score:							

**1.** Consider the polynomial  $P(x) = x^5 - 2x^4 - 4x^3 + 4x^2 - 5x + 6$ .

[6] (a) Use Descartes' rules of signs to state the number of possible positive and negative roots of P(x)

[4] (b) Use the Rational Root Theorem to list all possible rational roots of P(x).

[12] (c) Given that i is a root of P(x), find the other roots of this polynomial.

[5] (d) Express P(x) as the product of linear factors only.

- **2.** Consider the vectors  $\vec{v_1} = a\vec{i} + 2\vec{j} + 4\vec{k}$  and  $\vec{v_2} = a\vec{i} + 2\vec{j} + a\vec{k}$ , where *a* is a real number.
- [7] (a) Determine for which values of a the vectors  $\vec{v}_1$  and  $\vec{v}_2$  are perpendicular.

[10] (b) Determine the values a for which  $\vec{v}_1 \times \vec{v}_2 = \vec{0}$ .

- **3.** Let  $\mathcal{P}$  denote the plane that is determined by the points  $P_1(1,1,0), P_2(1,0,1)$  and  $P_3(0,1,1)$ . Find
- [12] (a) the point-normal equation for  $\mathcal{P}$ .

[5] (b) the equations in symmetric form of the line which is perpendicular to the plane  $\mathcal{P}$  and contains the point  $P_1$ .

**4.** Given the matrix  $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$ , indicate whether or not the expressions below are defined. If an expression is defined, evaluate the resulting matrix; if it is not defined, explain why not.

[2] (a)  $A^T$ .

[3] (b)  $A + A^T$ .

[8] (c)  $AA^T$ .

5. Consider the system of linear equations.

2x + 6w = 2; z + 7w = 1; 2x + y + 3w = 1; 4x + 15w = 1.

[4] (a) Write down the augmented matrix of this system.

(b) Solve the system by using Gauss-Jordan elimination. Clearly indicate the elementary row operations that you use.

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[7] **6.** Evaluate

$$\det \left( \begin{array}{rrrr} 2 & 0 & 0 & 6 \\ 0 & 0 & 1 & 7 \\ 2 & 1 & 0 & 3 \\ 4 & 0 & 0 & 15 \end{array} \right).$$

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