THE UNIVERSITY OF MANITOBA

DATE: December 8, 2012 (6:00 p.m. – 8:00 p.m.) **FINAL EXAMINATION**

DEPARTMENT & COURSE NO: MATH1210 **TIME**: 2 hours

EXAMINATION: Techniques of Classical and Linear Algebra **EXAMINERS**: D. Kalajdzievska, S. Kalajdzievski, D. Trim

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INSTRUCTIONS:

1. No aids permitted.
2. Attempt all questions.
3. If insufficient space is provided for a solution to a problem, continue your work on the back of the previous page.
4. Check that your examination booklet contains questions numbered from 1 to 10.
5. Fill in the information requested below.

Student Name (Print): ______________________________

Student Signature: ______________________________

Student Number: ______________________________

Seat Number: ______________________________

Circle your instructor's name: D. Kalajdzievska S. Kalajdzievski D. Trim

Question	Maximum Mark	Assigned Mark	Question	Maximum Mark	Assigned Mark
1	6		6	5	
2	5		7	8	
3	6		8	4	
4	5		9	10	
5	3		10	8	
Total	25		Total	35	

Examination Total /60

6 **1.** (a) According to the rational root theorem, what are the possible rational numbers that can satisfy the equation

$$5x^3 + 4x^2 + 4x - 1 = 0?$$

(b) Find all roots of the equation.

5 **2.** Find parametric equations for the line through the point $(2, -1, 3)$ parallel to the line defined by

$$x + y - 2z = 4, \quad 2x - y + 3z = 6.$$

6 **3.** Find all solutions of the system of equations

$$\begin{aligned} x + y + z &= 2, \\ 2x - y + 3z &= -1, \\ 7x + y + 9z &= 4. \end{aligned}$$

5 **4.** Use Cramer's rule to find the value of y satisfying the system of equations

$$\begin{aligned} x + 2y - 3z &= 4, \\ 2x + y &= 3, \\ x - y + z &= -1. \end{aligned}$$

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3 **5.** You are given that the inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{pmatrix} \qquad \text{is} \qquad A^{-1} = \frac{1}{8}\begin{pmatrix} 13 & -4 & -1 \\ -15 & 4 & 3 \\ 10 & 0 & -2 \end{pmatrix}.$$

Use the inverse matrix to find the value of z satisfying the system of equations

$$\begin{aligned} x + y + z &= 3, \\ 2y + 3z &= -4, \\ 5x + 5y + z &= 10. \end{aligned}$$

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5 **6.** Determine whether the following sets of vectors are linearly dependent or linearly independent. Justify your answers.

(a) $\langle 1, 2, -3\rangle$, $\langle 4, 2, 1\rangle$, $\langle -5, 1, 17\rangle$, $\langle 4, 1, 6\rangle$

(b) $\langle 1, 3, -2\rangle$, $\langle 2, 4, 5\rangle$, $\langle -2, 1, 3\rangle$

8 **7.** The matrix of a linear transformation T is

$$A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}.$$

(a) Find $T\langle 1, -2, 4\rangle$.

(b) Find all eigenvalues of the linear transformation.

(c) Find all eigenvectors associated with the smallest eigenvalue in part (b).

4 **8.** (a) Show that if λ is an eigenvalue of a matrix A, then λ^2 is an eigenvalue for the matrix A^2.

(b) If $\mathbf{v}$ is an eigenvector corresponding to λ, what is an eigenvector corresponding to λ^2?

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10 **9.** In the five parts of this question, circle the correct answer.

(a) When the determinant of the coefficient matrix of a system of n linear, homogeneous equations in n unknowns has value zero, the system has

(i) one solution
(ii) no solutions
(iii) an infinity of solutions

(b) The augmented matrix of a system of m equations in n unknowns has r leading ones in its RREF, where $r < n$. The system has

(i) no solutions
(ii) one solution
(iii) an infinity of solutions with $m - r$ parameters
(iv) an infinity of solutions with $n - r$ parameters
(v) an infinity of solutions with an undetermined number of parameters

(c) A system of n equations in m unknowns is known to have exactly one solution. The number of leading ones in the RREF of the augmented matrix for the system is

(i) n
(ii) m
(iii) $n - m$
(iv) $m - n$
(v) none of the above

(d) $(3AB)^T$ is equal to

(i) $3A^TB^T$
(ii) $3B^TA^T$
(iii) $(1/3)A^TB^T$
(iv) $(1/3)B^TA^T$
(v) none of the above

(e) If A is an invertible matrix, then

(i) $(A^2)^{-1} = (A^{-1})^2$
(ii) $(A^2)^{-1} = \dfrac{1}{A^2}$
(iii) A^2 does not have an inverse
(iv) A^2 has an inverse but it is not (i) or (ii)

8 **10.** Use mathematical induction to prove that 576 divides $5^{2n+2} - 24n - 25$ for $n \geq 1$.