

**MATH 1210    Assignment 1    Fall 2012**

1. (a) Use mathematical induction to prove that

$$1^2 + 4^2 + 7^2 + 10^2 + 13^2 + \cdots + (6n + 1)^2 = (2n + 1)(12n^2 + 9n + 1), \quad n \geq 1.$$

- (b) You were able to prove that the sum of the terms on the left side of the equation in part (a) is given by the formula on the right because we gave the formula to you. Suppose now that the formula is not given to you, but you have to find it. Do so using sigma notation.

2. Prove that  $3^n + 7^n - 2$  is divisible by 8 for all  $n \geq 1$ .

3. (a) Use mathematical induction to prove that  $n^2 > 2n + 1$  for all  $n \geq 3$ .  
(b) Use the result in part (a) to prove that  $2^n > n^2$  for  $n \geq 5$ .

4. Find a formula for

$$\sum_{i=1}^n \frac{1}{4i^2 - 1}.$$

Hint: See Exercise 34 in Section 1.2.

5. Evaluate

$$\sum_{j=m}^n (2j - 1)^2$$

for  $n > m$ . It is not necessary for you to simplify your result.

6. Simplify each of the following complex expressions to Cartesian form:

$$(a) \quad \frac{i^{15}(3 + 2i)^2}{1 - i} \qquad (b) \quad \frac{i^6(1 + 2\bar{i})(\overline{3 - 4i})}{3 + \overline{2i}}$$

7. Find all solutions for each of the following equations:

$$(a) \quad 2z^2 + 3z + 15 = 0 \qquad (b) \quad z^4 + 3z^2 - 5 = 0$$