MATH 1210 Assignment 3 Fall 2012

1. Given the vectors $\mathbf{u} = \langle 3, 1\rangle$, $\mathbf{v} = \langle 2, -4\rangle$, and $\mathbf{w} = \langle 5, -2\rangle$, find scalars a and b such that $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$.

2. Show that the lines

$$\begin{aligned} x &= 1 - t, \\ y &= -3 + 3t, \\ z &= 2 + 4t, \end{aligned} \qquad \text{and} \qquad \begin{aligned} x &= 3 + 5s/2, \\ y &= 2 + 7s/2, \\ z &= 5 + s, \end{aligned}$$

intersect, and find the acute angle between them.

3. Find the components of all vectors $\mathbf{v}$ which have length 2 and are perpendicular to both the lines $x = 4 + 3t$, $y = 2 - t$, $z = 1 + 5t$ and $x - y + z = 2$, $3x + 2y - 4z = 6$.

4. Find the equation of the plane, simplified as much as possible, that contains the point where the line $x = -1 + 2t$, $y = -4 + 2t$, $z = 1 - 4t$ intersects the xz-plane, and is perpendicular to the line

$$\frac{x+1}{3} = \frac{3y+1}{6} = \frac{1-2z}{4}.$$

5. Find, if possible, the equation of a plane containing the two lines

$$\begin{aligned} x - y + 2z &= 9, \\ 2x + y - 3z &= -9, \end{aligned} \qquad \text{and} \qquad \begin{aligned} 2x + y - 4z &= -12, \\ x + 3y + 5z &= 10. \end{aligned}$$

6. (a) Prove that if A, B, and C are three points in space, then the area of triangle ABC can be calculated with the formula

$$\text{Area of } \Delta\, ABC = \frac{1}{2}|\mathbf{AB} \times \mathbf{AC}|.$$

(b) Use the formula in part (a) to find the area of the triangle with vertices $(2, 0, -3)$, $(1, 5, 6)$, and $(-1, 3, 4)$.