Midterm Examination for Math 1210 **60 minutes**

INSTRUCTIONS:

1. No aids permitted.
2. Attempt all questions.
3. If insufficient space is provided for a solution to a problem, continue your work on the blank page at end of the examination. Clearly indicate that you have done so. Do not remove this page.
4. Check that your examination booklet contains ten questions.
5. Fill in the information requested below.

Student Name (Print): _____

Student Signature: _____

Student Number: _____

Circle your instructor's name: D. Kalajdzievska S. Kalajdzievski D. Trim

Question	Maximum Mark	Assigned Mark	Question	Maximum Mark	Assigned Mark
1	10		6	4	
2	6		7	4	
3	5		8	4	
4	6		9	3	
5	3		10	3	
Total	30		Total	18	

Examination Total /48

10 **1.** Use mathematical induction to prove that

$$1 + 2(2) + 3(2^2) + 4(2^3) + \cdots + n(2^{n-1}) = 1 + (n+1)2^n - 2^{n+1}.$$

6 **2.** Evaluate the summation

$$\sum_{i=20}^{45} (4i^2 - 1).$$

Do not simplify your answer. You may use any of the following formulas,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

5 **3.** Find the imaginary part of the complex number

$$(1 - \sqrt{3}i)^{10},$$

simplified as much as possible.

6 **4.** Find all solutions of the equation

$$x^4 + 1 = 0.$$

Express any complex solutions in Cartesian form, simplified as much as possible.

3 **5.** If A and B are the matrices

$$A = \begin{pmatrix} 3 & 2 & 0 \\ -1 & 4 & 2 \\ 0 & 1 & 5 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 6 & 7 \\ 2 & 1 & 5 \end{pmatrix},$$

what is the $(2, 3)$ entry of $A^2 + 2B^T$; that is, the entry in the second row and third column.

4 **6.** Find a vector of length 3 that is perpendicular to both the y-axis and the vector $\langle 3, -1, 2\rangle$.

4 **7.** Find the equation of the plane that passes through the point $(-1, 2, 4)$ and is parallel to the plane $8x - 4y + 2z = 11$. Simplify the equation as much as possible.

4 **8.** What do Descartes' rules of sign predict for the number of positive and negative roots of the equation

$$2x^5 - 3x^3 - 5x = -1?$$

3 **9.** If x is a complex number that satisfies the equation

$$3x^5 + 2x^4 - 10x^2 + 5 = 0,$$

what does the bounds theorem predict about the modulus of x?

3 **10.** According to the rational root theorem, what are the possible rational numbers that can satisfy the equation

$$4x^3 - 4x^2 + 5x + 10 = 0?$$