UNIVERSITY OF MANITOBA

DATE: October 24, 2013 — MIDTERM TITLE PAGE

EXAMINATION: Techniques of Classical and Linear Algebra — TIME: 1 hour

COURSE: MATH 1210 — EXAMINER: Davidson, Harland, Moghaddam

NAME: (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

❑	A01	9:30–10:20 AM	MWF (200 Armes)	M. Davidson
❑	A02	1:30–2:20 PM	MWF (204 Armes)	G. I. Moghaddam
❑	A03	1:30–2:20 PM	MWF (100 St. Paul)	N. Harland

INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 55 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	11	
2	6	
3	16	
4	7	
5	8	
6	7	
Total:	55	

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[11] 1. (a) Use mathematical induction on integer $n \geq 1$ to prove that

$$2 + 5 + 8 + \ldots + (6n - 1) = n(6n + 1)\,.$$

(b) Write $2 + 5 + 8 + \ldots + (6n - 1)$ in sigma notation.

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[6] 2. Find all fourth roots of $-2 - 2\sqrt{3}\,i$. Leave your answer in exponential form.

[16] 3. Consider the polynomial equation of $P(x) = 0$ where

$$P(x) = 3x^4 - 8x^3 + 4x^2 + 25$$

(a) What are the possible rational zeros of $P(x)$?

(b) Use Descartes' rule of signs to find the possible number of positive and negative roots of $P(x)$.

(c) Use Bounds Theorem to find a bound on the roots of $P(x)$.

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$[\textit{Recall that } P(x) = 3x^4 - 8x^3 + 4x^2 + 25\,]$

(d) Update the list from part (a) using the information from parts (b) and (c).

(e) Given that $2 + i$ is a root of $P(x)$, find all the roots of $P(x)$.

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[7] 4. Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ -1 & 0 & 1 & 0 & -1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$. Evaluate each of the following expressions or explain why it is not defined.

(a) $B(A - A^T)$.

(b) $(B + B^T)A$.

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[8] 5. Let $\mathbf{u} =< a, b, c >$, $\mathbf{v} =< 1, 2, -1 >$ and $\mathbf{w} =< 3, 1, 5 >$.

(a) Find all values of a for which $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = b^2 + c^2$.

(b) Find the angle between $-\mathbf{2v}$ and $\mathbf{3w}$.

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[7] 6. Consider the point $P(4, 1, -3)$ and the two lines $L_1 : \ x = t\,,\ y = 1 + 2t\,,\ z = 1$ and $L_2 : \ x = -r\,,\ y = 6 + 3r\,,\ z = 2 + r\,.$

(a) Find parametric equations of the line through the point P and perpendicular to both lines L_1 and $L_2\,.$

(b) Find an equation of the plane through the point P and parallel to both lines L_1 and $L_2\,.$
