UNIVERSITY OF MANITOBA

DATE: December 18, 2014

COURSE: MATH 1210

EXAMINATION:

Techniques of Classical and Linear Algebra

FINAL EXAMINATION

TITLE PAGE

TIME: 120 Minutes

EXAMINER: Various

NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________

SIGNATURE: (in ink) ______________________________

(I understand that cheating is a serious offense.)

- ❑ A01 9:30-10:20 MWF J. Arino
- ❑ A02 13:30–14:20 MWF X. Zhao
- ❑ A03 13:30–14:20 MWF J. Arino

INSTRUCTIONS TO STUDENTS:

This is a 120 Minute exam. **Please show your work clearly.**

No texts, notes, calculators, cellphones, translators or any other electronic devices are permitted.

This exam has a title page, 11 pages of questions (including 1 blank page for rough work). Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staples.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued and label the continuation.

Question	Points	Score
1	13	
2	6	
3	10	
4	15	
5	11	
6	10	
7	6	
8	6	
9	10	
10	8	
11	5	
Total:	100	

[8] 1. (a) Use mathematical induction to show that for all integers $n \geq 1$,

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}. \tag{1}$$

[5] (b) Use Identity (1) and other known identities to compute

$$\sum_{k=1}^{10} \left(2k^2 - k + 1\right).$$

[6] 2. Find all solutions of the equation $z^2 = 2 + 2i$. Express your answers in exponential form.

3. Consider the polynomial $f(x) = -x^{2014} - x^{2013} - x - 1$.

[5] (a) Find the remainder when $f(x)$ is divided by $x + i$, where i is the fundamental complex number.

[5] (b) Show that $f(x)$ has no zeros greater than 1.

4. Let $P_1 : 2x + 3y - z = 1$ and $P_2 : x + 2y + 3z = 2$ be two planes, $L_1 : (x, y, z) = (1, 0, 2) + t(1, 2, 3)$, $t \in \mathbb{R}$, be a line and $A(2, 3, 1)$ be a point.

[5] (a) Find the intersection of the planes P_1 and P_2.

[5] (b) Find the equation of the line L_2 parallel to L_1 and going through the point A.

[5] (c) Find the intersection of L_1 and P_2. [Hint: you are looking for a value of t such that a point (x, y, z) on L_1 satisfies P_2.]

5. Let $A = \begin{bmatrix} -1 & -5 & 7 \\ 0 & 1 & -4 \\ 0 & 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & -2 & -4 \end{bmatrix}$, and C be an arbitrary invertible 3×3 matrix.

[5] (a) Find $\det(A^2 C^3 (B^T)^{-1} C^{-3})$.

[6] (b) Find $\det(\operatorname{adj} A)$. (Hint: you do not need to compute $\operatorname{adj} A$. Recall the formula involving A^{-1} and $\operatorname{adj} A$.)

[6] 6. (a) Use direct method to find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 1 & -1 & 10 \end{bmatrix}$. Indicate clearly the elementary row operations used in each step.

[4] (b) Use the information from part (a) to solve the following system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 2 \\ 2x_1 + 5x_2 + 4x_3 &= 0 \\ x_1 - x_2 + 10x_3 &= 15 \end{aligned}$$

[6] 7. Show that the following vectors are linearly independent:

$$\langle 2,3,4\rangle, \langle 1,2,-1\rangle, \langle 3,-5,0\rangle.$$

[6] 8. Let A be a square matrix. State three additional properties that are equivalent to "A is invertible".

(1)____

(2)____

(3)____

9. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & -3 & 0 \end{bmatrix}$.

[5] (a) Find all eigenvalues for A.

[5] (b) Find the eigenvector(s) corresponding to the eigenvalue $\lambda = 1$.

10. A square matrix $A \in \mathcal{M}_n$ is *idempotent* if $A^2 = A$.

[4] (a) Show that the determinant of an idempotent matrix can only be 0 or 1.

[4] (b) Show that the only nonsingular idempotent matrix is the identity matrix I_n.

[5] 11. Let A be a square matrix. Assume that (λ, v) is an eigenpair of A. Use mathematical induction to show that (λ^n, v) is then an eigenpair of A^n, for any integer $n \geq 2$.

Blank page: For rough work only; no work on this page will be marked.