Attempt all questions and show all your work. Due September 25, 2015.

1. Use Mathematical Induction to prove that for all $n \geq 1$,
$$n + (n+1) + (n+2) + \cdots + (2n) = \frac{3n(n+1)}{2}.$$

2. Use Mathematical Induction to prove that for all $n \geq 1$,
$$\sum_{i=1}^{n} (i+3)^2 = \frac{n(2n^2+21n+73)}{6}$$

3. Use Mathematical Induction to prove that for all $n \geq 1$,
$$1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^{2n}} = \frac{3}{2}\left(1 - \left(\frac{1}{3}\right)^{2n+1}\right).$$

4. (a) Write the sum $1 + 3 + 5 + \cdots + (4n-1)$ using sigma notation.
   (b) Use Mathematical Induction to prove that for all $n \geq 1$, the above expression is equal to $(2n)^2$.

5. Use Mathematical Induction to prove that for all $n \geq 1$, $3^n > n^2$.

6. Consider the sequence of real numbers defined by the relations $x_0 = 0.5$ and for all $n \geq 1$, $x_n = 0.5x_{n-1}(1 - x_{n-1})$. Prove by induction that for all $n \geq 0$, $x_n \in (0, 1)$.

7. Let a and d be fixed real numbers. Prove using Mathematical Induction that for each $n \geq 1$,
$$a + (a+d) + (a+2d) + \cdots + (a+(n-1)d) = \frac{n}{2}(2a + (n-1)d).$$

8. (a) Express the sum $\displaystyle\sum_{k=1}^{m} (2+3k)^2$ in terms of three simpler sums in sigma notation by expanding. Do not calculate the value.
   (b) Find the value of the sum
$$\sum_{p=1}^{100} (2 - 10p + 3p^2).$$
   HINT: Make use of the formulas
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \text{and} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$
   (c) Rewrite the sum
$$\sum_{r=12}^{122} \frac{r-6}{r+9}$$
   using an index whose initial and terminal values are 1 and 111 (HINT: use a change of variables).