MATH 1210 Assignment 2 16R-T1

Attempt all questions and show all your work. Due October 9, 2015.

1. Simplify $\frac{169}{5+12i} + \left(\overline{(1-2i)^3+4}\right)^2$ and express in Cartesian form.

2. Express in the forms required, with all arguments in your answers reduced to numbers in the interval $(-\pi, \pi]$.
   (a) $-6 + i\sqrt{108}$ in polar and exponential forms
   (b) $\sqrt{18}\left(\cos\frac{19\pi}{4} + i\sin\frac{19\pi}{4}\right)$ in Cartesian and exponential forms
   (c) $10e^{\frac{-5\pi}{6}}$ in Cartesian and polar forms

3. $\cos n\theta$, $n \in \mathbb{Z}$, can always be expressed in terms of $\sin\theta$ and $\cos\theta$. For example, $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$. Use De Moivre's Theorem to obtain an expression of this type for $\cos 7\theta$.

4. Find all of the complex 6th roots of -64. Express your answers in Cartesian form.

5. Solve the equation $x^4 - 8x^2 + 36 = 0$ over the complex numbers.

6. (a) Use long division to find the quotient and remainder when $x^5 - 3x^4 + 2x^2 - x + 7$ is divided by $x - 3$. Express the result as an equation of the form

$$\text{(polynomial)} = \text{(polynomial)} \cdot \text{(quotient)} + \text{(remainder)}.$$

   (b) Use the Remainder Theorem to find the remainder when

$$f(x) = (1+i)x^4 + 3ix^3 + (1-i)x + 2$$

   is divided by $ix - 3$ (Do not perform long division!)

   (c) For which value of d is the polynomial $2x - 3$ a factor of the polynomial $g(x) = x^3 - 5x^2 + 2x - d$?

   (d) You are given that $(x-2)$ and $(x+1)$ are factors of the polynomial $f(x) = x^4 - 8x^3 + hx^2 + kx + 6$. Find h and k.

7. You are given that $2 + i$ is a zero of the polynomial $p(x) = x^4 - 4x^3 + 9x^2 - 16x + 20$. Write $p(x)$ as a product of linear factors. What are the roots of the equation $p(x) = 0$?

8. In each case your response should refer by number to appropriate results in the textbook as needed.
   (a) If a polynomial of degree n with real coefficients does not have n real zeros (counting multiplicity) then it must have an irreducible quadratic factor. Justify this statement.
   (b) If r is a zero of a polynomial $f(x)$ of multiplicity 5 and a zero of the polynomial $g(x)$ of multiplicity 7, must it also be a zero of the polynomial $h(x) = f(x) + g(x)$? If so, can we determine its multiplicity? If so, what is it? If not, why not?