Attempt all questions and show all your work. Due October 30, 2015.

1. Simplify each of the following matrix expressions, or state why it can't be done.
   (a) $\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T \begin{bmatrix} -1 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix}\right)^T$
   (b) $\left((I_2)^2 + (I_3)^2\right)^T$
   (c) $3I_3^2 - (5I_3)^T + \left((2I_3)^2\right)^T$.

2. A linear system of equations has 4 variables, a, b, c, and d. The RREF of the augmented matrix for this system is
$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & -1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$
   Find three different solutions for this system.

3. Solve the following system using **the strict textbook version** of Gaussian elimination:
$$\begin{array}{rcrcrcr} & & 2x_2 & - & x_3 & = & -1 \\ 3x_1 & - & 2x_2 & + & x_3 & = & 4 \\ 3x_1 & + & 2x_2 & + & x_3 & = & -4 \end{array}$$

4. Solve the following systems by putting their augmented matrices into RREF.
   (a)
$$\begin{array}{rcrcrcr} x_1 & - & 3x_2 & & & = & -5 \\ & & x_2 & + & 3x_3 & = & -1 \\ 2x_1 & - & 10x_2 & + & 2x_3 & = & -20 \end{array}$$
   (b)
$$\begin{bmatrix} 3 & 3 & 1 \\ 3 & 2 & 2 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}$$

5. Find all values for a and b such that $\begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -a & a \\ b & -b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

6. Find $a, b, c \in \mathbb{R}$ such that
$$a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c \begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 22 \\ -13 \\ -1 \end{bmatrix}.$$
   Use any method and show all your work.

7. Consider the following augmented matrix for a linear system of equations:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

Find **all values** for a that will result in this system having **infinitely many solutions**. Justify your answer.

8. Let $A = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$. Find all values for k such that $(kA)^T(kA) = \begin{bmatrix} 1 \end{bmatrix}$.

9. Find an example of two matrices A and B such that $(AB)^T \neq A^T B^T$. Justify your answer.

10. What can be said about the number of solutions to a system of equations given that the RREF of the coefficient matrix contains a zero row? Explain and justify your answer as appropriate.

11. Let A and B be two matrices and let $r \in \mathbb{R}$, $r \neq 0$. Prove that if $rA = rB$, then $A = B$.

12. Let A be a matrix, $\mathbf{b}$ a column matrix. Let $\mathbf{x}_0$ and $\mathbf{x}_1$ both be solutions to the system of equations $A\mathbf{x} = \mathbf{b}$. Prove that $\mathbf{x}_0 - \mathbf{x}_1$ is a solution to the homogeneous system of equations $A\mathbf{x} = \mathbf{0}$.

13. Show that any two matrices which commute with

$$M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

also commute with each other.