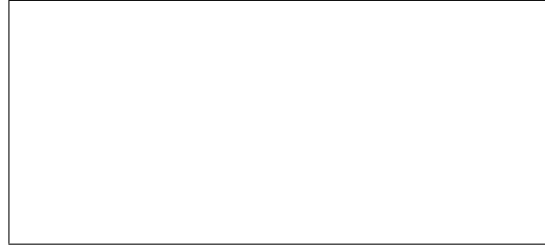


Unique Identifier Sticker:

**DO NOT WRITE ABOVE THIS LINE.** _____**INSTRUCTIONS TO STUDENTS:**

This is a 60 minute exam. **Please show your work clearly.**

No texts or notes are permitted. No calculators are permitted. Cell phones, electronic translators, and other electronic devices are **not** permitted.

This exam has a title page and 8 pages of questions, including 2 blank pages for rough/extra work. Please check that you have all the pages.

The value of each question is indicated beside the statement of the question. The total value of all questions is 73 points.

If you need more scrap paper, use the back of the question pages. **Anything written on the back of a page will not be marked.** If you need more space to answer a question (that you want marked), write it on one of the scrap pages at the back.

INDICATE YOUR SECTION:

- A01 (9:30-10:20 MWF) R. Borgersen
- A02 (13:30-14:20 MWF) J. Arino
- A03 (13:30-14:20 MWF) R. Borgersen

FIRST NAME: _____

LAST NAME: _____

STUDENT NUMBER: _____

PAPER NUMBER: _____

(Paper Number is found at the top)

SIGNATURE: (in ink)

(I understand that cheating is a serious offense)

UNIVERSITY OF MANITOBA

DATE: October 20, 2015

MIDTERM EXAMINATION

PAGE: 1 of 8

DEPARTMENT & COURSE NO: MATH 1210

TIME: 60 minutes

EXAMINATION: Techniques of Classical and Linear Algebra

EXAMINER: Borgersen/Arino

1. [15 points] Prove by induction that for all $n \geq 1$, $\sum_{i=1}^{2n} 3i - 2 = n(6n - 1)$. Show all your work, use complete sentences, and use the style done in class.

2. (a) [3 points] For any integer k , we introduce the notation $k! = k \cdot (k - 1)(k - 2) \cdots 2$, which we call "factorial k ". Using this, write

$$1 + \frac{1}{2} + \frac{1}{3 \cdot 2} + \frac{1}{4 \cdot 3 \cdot 2} + \cdots + \frac{1}{n \cdot (n - 1) \cdots 2}$$

in sigma notation (do not evaluate).

3. Write the following sums using sigma notation with indexes starting at 1 (do not evaluate):

(a) [3 points] $\frac{1}{6} + \frac{1}{3} + \frac{2}{3} + \frac{4}{3} + \frac{8}{3} + \cdots + \frac{64}{3}$

(b) [4 points] $-x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120}$

4. Let $f(x) = x^5 - 2x^3 + 2x^2 - 3x + 2$. Then $f(-x) = -x^5 + 2x^3 + 2x^2 + 3x + 2$.

(a) [4 points] Given that i is a root of $f(x)$, write $f(x)$ in the form $f(x) = P_2(x)Q_3(x)$, where $P_2(x)$ is an irreducible quadratic form and $Q_3(x)$ is a cubic polynomial.

(b) [4 points] What do Descartes' Rules of Signs say about $f(x)$? Be specific, and use complete sentences.

(c) [3 points] What does the Bounds Theorem say about $f(x)$? Be specific, and use complete sentences.

(d) [6 points] Taking the results of (a), (b) and (c) into account, use the rational root theorem to list all possible rational zeros of $f(x)$. Finish by listing all the zeros of $f(x)$ together with their multiplicity if it is not equal to 1.

5. [3 points] Let $g(x) = 5x^5 + 4x^4 + kx^3 + 2x^2 + x + 1$. For what value(s) of k will $g(x)$ have remainder 15 when divided by $x - 1$?

6. (a) [3 points] Express $z_1 = \sqrt{2} + \sqrt{2}i$ in exponential form:

(b) [3 points] Express $z_2 = 6 \left(\cos \left(\frac{5\pi}{6} \right) + i \sin \left(\frac{5\pi}{6} \right) \right)$ in Cartesian form.

(c) [3 points] Express $z_3 = -7e^{\frac{\pi}{3}i}$ in polar form.

7. Consider the complex number $z = 27(1 - i)$.

(a) [3 points] Write z in polar and exponential forms.

(b) [6 points] Find all cube roots of z . You can leave your answers in exponential form.

8. Consider the following matrices:

$$A = \begin{bmatrix} -4 & -1 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} -9 & -1 & 0 \\ 0 & 6 & -5 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & -2 & -5 \\ 7 & 8 & 2 \\ 8 & -5 & -3 \end{bmatrix},$$

$$D = \begin{bmatrix} 6 & 0 & -3 \\ 4 & 2 & -5 \\ 3 & -1 & -6 \\ 5 & 1 & -2 \end{bmatrix}, \quad E = \begin{bmatrix} 4 & -4 \\ -4 & -1 \\ 0 & -8 \\ -2 & 7 \end{bmatrix}, \quad F = \begin{bmatrix} -4 & -5 \\ -4 & 8 \\ -10 & 1 \end{bmatrix}.$$

- (a) [8 points] For each expression below, indicate with an "x" in the appropriate column if it is undefined or defined, and if it is defined, indicate the size of the resulting matrix. Do not compute the resulting matrix.

Expression	Undefined	Defined	Size
ACF			
$F^T F + I_2$			
$D^T E - B$			
$AA^T - A^T A$			
$DC^T - D$			

- (b) [2 points] Evaluate the (2, 3) entry of $B^T F^T - C$ if it is defined. If it is not defined, explain why.

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