UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS

MATH 1210 Techniques of Classical and Linear Algebra

FINAL EXAMINATION

January 11, 2017, 7:30–9:30 PM

LAST NAME: ______________________________

FIRST NAME: ______________________________

STUDENT NUMBER: ______________________________

This exam is intended for students in section A02. Students who have written an exam for this course in December will not be given credit if they write this exam as well.

SIGNATURE: ______________________________
(I understand that cheating is a serious offense and I have read and understand the above)

Please indicate your instructor and section by checking the appropriate box below:

- ☐ A01 MWF (9:30 am – 10:20 am) W. Grafton
- ☐ A02 MWF (13:30 pm – 14:20pm) G.I. Moghaddam
- ☐ A03 MWF (13:30 pm – 14:20pm) M. Szestopalow

INSTRUCTIONS TO STUDENTS:

Fill in clearly *all the information above.*

This is a 120 minute exam.

No *calculators, texts, notes, cellphones or other aids are permitted.*

Show your work clearly *for full marks.*

This exam has a title page, 10 pages of questions and 1 blank page at the end for rough work. Please check that you have all pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 90.

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the ***reverse*** *side of the page, but* ***clearly indicate*** *that your work is continued there.*

DO NOT WRITE IN THIS TABLE

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	7	9	10	11	6	8	10	10	8	11	90
Score:											

---

[7] **1.** Let $A = \begin{bmatrix} i & i \\ 0 & i \end{bmatrix}$ where i is the complex number for which $i^2 = -1$. Use mathematical induction to prove that

$$A^{(2n)} = \begin{bmatrix} (-1)^n & (-1)^n(2n) \\ 0 & (-1)^n \end{bmatrix}$$

for all positive integers $n \geq 1$.

---

[9] **2.** Consider the polynomial equation of $P(x) = 0$ where

$$P(x) = x^4 - 2x^3 - x + 2\,.$$

(a) What are the number of the possible positive and negative zeros of $P(x)$?

(b) Find all the zeros of $P(x)$.

[10] **3.** Consider the two planes $\Pi_1;\ 3x + 2y + 3z = 1$ and $\Pi_2;\ 7x + 5y + 9z = 4$.

(a) Is the plane Π_1 perpendicular to the plane Π_2 ? Why?

(b) Find parametric equations of the line through the point $(2, 1, 0)$ and parallel to the line $3x + 2y + 3z = 1$, $7x + 5y + 9z = 4$.

[11] **4.** For the following homogeneous linear system of equations, first find reduced row echelon form of the augmented matrix and then find all ***basic*** solutions of the system.

$$\begin{aligned}
5x_1 + 10x_2 + 2x_3 \qquad\qquad & -7x_5 + x_6 = 0 \\
2x_1 + 4x_2 + x_3 \qquad\qquad & -3x_5 + x_6 = 0 \\
7x_1 + 14x_2 \qquad + 2x_4 & -9x_5 - x_6 = 0
\end{aligned}$$

---

[6] **5.** Let $A = \begin{bmatrix} 8a & -1 & 0 \\ -1 & 1 & -1 \\ -a & -2a & 3a \end{bmatrix}$. Find all values of "$a$" for which the matrix A is invertible.

---

[8] **6.** Use Cramer's rule to solve the following linear system of equations for z ***only***.
(Do **not** solve it for x, y and u)

$$
\begin{aligned}
x + 3y - u &= 4 \\
7x + y &= 2 \\
5x - z &= 0 \\
-x + 2y + 4z &= 1
\end{aligned}
$$

[10] **7.** Let $A = \begin{bmatrix} 1 & 4 & 1 \\ 0 & 3 & 1 \\ -1 & -6 & -2 \end{bmatrix}$. First find A^{-1} and then use it to solve the linear system

$$A^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}.$$

---

[10] **8.** Let $\mathbf{u} =< 1, 4, 0, -1 >$, $\mathbf{v} =< 1, -6, 2, -1 >$, and $\mathbf{w} =< 0, 5, -1, 0 >$. First show that the vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are linearly dependent and then write $\mathbf{w}$ as a linear combination of $\mathbf{u}$, and $\mathbf{v}$.

---

[8] **9.** Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation such that $T(2\,\hat{\mathbf{i}}) = \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}$, $T(-\hat{\mathbf{j}}) = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, and $T(\frac{1}{3}\,\hat{\mathbf{k}}) = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. First find the matrix associated with T, and then use it to find the image of the vector $\mathbf{u} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ under T.

[11]**10.** Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$.

(a) Find all eigenvalues of A.

(b) Given that $\lambda = i$ is an eigenvalue of A, find all corresponding eigenvectors.

This page is intentionally left blank. Use it for rough work.