UNIVERSITY OF MANITOBA

DATE: October 24, 2016 MIDTERM
TITLE PAGE

EXAMINATION: Techniques of Classical and Linear Algebra TIME: 75 minutes
COURSE: MATH 1210 EXAMINER: Grafton, Moghaddam, Szestopalow

NAME: (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

❑	A01	9:30–10:20 AM	MWF (201 Armes)	W. Grafton
❑	A02	1:30–2:20 PM	MWF (221 Wallace)	G. I. Moghaddam
❑	A03	1:30–2:20 PM	MWF (206 Human Ecology)	M. Szestopalow

INSTRUCTIONS TO STUDENTS:

This is a 75 minutes exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 60 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	11	
2	10	
3	7	
4	16	
5	8	
6	8	
Total:	60	

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[8] 1. (a) Use mathematical induction on integer $n \geq 1$ to prove that

$$1(3) + 2(4) + 3(5) + \ldots + n(n+2) = \frac{1}{6}\, n(n+1)(2n+7)\,.$$

[3] (b) Write $1(3) + 2(4) + 3(5) + \ldots + n(n+2)$ in sigma notation such that the index starts from 2.

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[10] 2. Find the Cartesian form of $(i^5+1)^{10} + (i^5-1)^{10}$. Simplify as much as possible.

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[7] 3. Find all third roots of $-\sqrt{2}+\sqrt{6}\,i$. Leave your answer in exponential form but simplify it.

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[16] 4. Consider the polynomial equation of $P(x) = 0$ where

$$P(x) = 3x^4 - 6x^3 + 7x^2 - 8x + 4$$

(a) What are the possible rational zeros of $P(x)$?

(b) Show that $P(x)$ has no zero in the interval $[-3, -1]$.

(c) Use Bounds Theorem to find a bound on the zeros of $P(x)$.

⇒ Continued next page

[*Recall that* $P(x) = 3x^4 - 6x^3 + 7x^2 - 8x + 4$]

(d) Update the list from part (a) using the information from parts (b) and (c).

(e) Find all the zeros of $P(x)$.

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[8] 5. Let $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & -5 \end{pmatrix}$. Find all values of a and b for which $A^2 + AB^T - 6I = \mathbf{0}$.

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6. Let $\mathbf{u} =< 1, 3, 3 >$, $\mathbf{v} =< 1, 2, 2 >$ and $\mathbf{w} =< 1, 1, -\frac{1}{2} >$.

[4] (a) Find the angle between $\mathbf{u} \times \mathbf{v}$ and $\mathbf{2w}$.

[4] (b) Let $\mathbf{r} =< 1, 2a - 12, a^2 >$. Find all values of "$a$" for which $\mathbf{r}$ is perpendicular to $\mathbf{u} + \mathbf{v} - 2\mathbf{w}$.

BLANK PAGE FOR ROUGH WORK