UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS
MATH 1210 Techniques of Classical and Linear Algebra
FINAL EXAMINATION
December 16, 2017, 1:30–3:30 PM

LAST NAME: ______________________________

FIRST NAME: ______________________________

STUDENT NUMBER: ______________________________

SIGNATURE: ______________________________
(I understand that cheating is a serious offense
and I have read and understand the above)

Please indicate your instructor and section by checking the appropriate box below:

☐	A01	MWF (9:30 – 10:20 AM)	M. Szestopalow
☐	A02	MWF (1:30 – 2:20 PM)	G.I. Moghaddam
☐	A03	MWF (1:30 – 2:20 PM)	C. Ramsey

INSTRUCTIONS TO STUDENTS:

Fill in clearly *all the information above.*

This is a 120 minute exam.

No *calculators, texts, notes, cellphones or other aids are permitted.*

Show your work clearly *for full marks.*

This exam has a title page, 10 pages of questions and 1 blank page at the end for rough work. Please check that you have all pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 100.

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the ***reverse*** *side of the page, but* ***clearly indicate*** *that your work is continued there.*

DO NOT WRITE IN THIS TABLE

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	9	8	12	10	10	6	17	7	10	11	100
Score:											

---

[3] **1.** (a) Write $\big(1 + 6(1^2)\big) + \big(1 + 6(2^2)\big) + \big(1 + 6(3^2)\big) + \ldots + \big(1 + (6n^2 - 12n + 6)\big)$ in sigma notation.

[6] (b) Evaluate the sum $\displaystyle\sum_{j=10}^{18}[4(j-6)^3 + 10]$ using any of the following identities that you may find relevant.

$$\sum_{k=1}^{m} k = \frac{1}{2}\,[m(m+1)] \quad , \sum_{k=1}^{m} k^2 = \frac{1}{6}\,[m(m+1)(2m+1)] \quad , \sum_{k=1}^{m} k^3 = \frac{1}{4}\,[m^2(m+1)^2]$$

[8] **2.** Find all of the third roots of -125 in Cartesian form and simplify as much as possible.

3. Let $P(x) = x^4 - x^3 + x^2 + 6x - 4$.

[6] (a) Prove that $P(1+\sqrt{3}i) = 0$. Show your work.

[6] (b) Use part (a) to find all zeros of $P(x)$.

[10] **4.** Use Cramer's rule to find the solution of the system

$$\begin{array}{rrrcr} 5x & -2y & +z & = & 0 \\ -x & +2y & -z & = & -1 \\ & y & +2z & = & 0 \end{array}$$

[10] **5.** Find all **basic solutions** of the homogeneous system

$$\begin{array}{rrrrrl} 6x_1 & +3x_2 & +12x_3 & -3x_4 & & =0 \\ 5x_1 & +3x_2 & +11x_3 & -3x_4 & +2x_5 & =0 \\ 2x_1 & & +2x_3 & +x_4 & -7x_5 & =0 \end{array}$$

---

[6] **6.** Let $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & -2 & 4 \\ -1 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 2 \\ 1 & 1 & 4 \\ 0 & -2 & -4 \end{bmatrix}$. Determine whether the columns of the matrix AB are linearly independent or linearly dependent. Show your work.

7. Let $A = \begin{pmatrix} 5 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 5 & -3 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$.

[7] (a) Compute A^{-1}, if possible.

[7] (b) Compute B^{-1} using the Adjoint method.

[3] (c) Solve either $A\mathbf{x} = \mathbf{b}$ or $B\mathbf{x} = \mathbf{b}$. Do **NOT** solve both.

[7] **8.** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the function defined by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy + z \\ x \\ y \end{pmatrix}.$$

Prove that T is **not** a linear transformation. Show your work.

9. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotates vectors $\frac{3\pi}{4}$ radians counter clockwise about the origin and then reflects the resulting vector across the x-axis.

[8] (a) Find the matrix associated with T.

[2] (b) What is the result of applying the linear transformation T^{-1} to the vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$?

10. Let $A = \begin{pmatrix} 2 & 0 & 0 \\ -2 & -2 & 2 \\ -5 & -10 & 7 \end{pmatrix}$.

[8] (a) Find the eigenvalues of A. For each eigenvalue λ, find an eigenvector that corresponds to λ.

[3] (b) Let $\mathbf{v}$ be one of the eigenvectors from part (a). Compute $A^5\mathbf{v}$.

This page is intentionally left blank. Use it for rough work.