UNIVERSITY OF MANITOBA

DATE: October 26, 2017 MIDTERM
TITLE PAGE

EXAMINATION: Techniques of Classical and Linear Algebra TIME: 75 minutes
COURSE: MATH 1210 EXAMINER: Moghaddam, Ramsey, Szestopalow

NAME: (Print in ink) _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

STUDENT NUMBER: _____

❑	A01	9:30–10:20 AM	MWF (207 Buller)	M. Szestopalow
❑	A02	1:30–2:20 PM	MWF (100 St. Paul)	G. I. Moghaddam
❑	A03	1:30–2:20 PM	MWF (221 Wallace)	C. Ramsey

INSTRUCTIONS TO STUDENTS:

This is a 75 minutes exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 6 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 60 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	11	
2	10	
3	6	
4	15	
5	8	
6	10	
Total:	60	

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[8] 1. (a) Use mathematical induction on integer $n \geq 1$ to prove that

$$2!\,(2) + 3!\,(3) + 4!\,(4) + \ldots + (n+1)!\,(n+1) = (n+2)! - 2\,.$$

[3] (b) Write $2!\,(2) + 3!\,(3) + 4!\,(4) + \ldots + (n+1)!\,(n+1)$ in sigma notation such that the index starts from 0.

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[10] 2. Find the Cartesian form of $\dfrac{i^{62}\,(\sqrt{2}+\sqrt{6}\,i)^8}{2^8(-\sqrt{6}-\sqrt{2}\,i)}$. Simplify as much as possible.

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[6] 3. Find all of the fourth roots of $16i$. Leave your answers in exponential form, but simplify it.

4. Consider the polynomial equation $P(x) = 0$ where $P(x) = 2x^4+2x^3+x^2+5x-10$.

[3] (a) Use Rational Root Theorem to determine all possible rational roots of $P(x)$.

[3] (b) Let $Q(x) = (x-5)P(x)$. How many positive real roots and negative real roots does $Q(x)$ have? Explain.

[3] (c) Use Bounds Theorem to determine an upper bound for the modulus of roots of $P(x)$. Does this eliminate any possible rational roots? Which ones?

[6] (d) Find all of the solutions to $P(x) = 0$.

[8] 5. Let

$$A = \begin{bmatrix} 2 & k \\ 1 & 2 \\ 1 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 4 & 1 \\ 1 & 2 \\ -1 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} -1 & 1 \\ -4 & -3 \end{bmatrix}.$$

Find value(s) of k for which $B^T A - 2C^2 + I = D$.

6. Let $\mathbf{u} = \langle 1, 2, 4 \rangle, \mathbf{v} = \langle -1, 0, 1 \rangle$ and $\mathbf{w} = \langle 2, 1, -1 \rangle$.

[3] (a) Find the angle between the vectors $\mathbf{u}$ and $\mathbf{w}$.

[7] (b) Find a unit vector in the direction of the vector $\mathbf{r} = \big((\mathbf{u} + \mathbf{v}) \cdot \mathbf{w}\big)(\mathbf{v} - 2\mathbf{u})$.

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