MATH 1210 (FALL TERM 2019)
ASSIGNMENT ONE

Please complete the entire assignment, staple it to the Honesty Declaration Form (which should serve as the front page) and submit it to your instructor on October 4th, 2019 at the beginning of the class. Only some of the questions will be marked. Late assignments will not be accepted.

Q1. Let $P(n)$ denote the statement:

$$(n+3)+(n+4)+(n+5)+\cdots+(3n+1)=2\,(n+1)\,(2n-1). \tag{1}$$

(a) Check the statement $P(2)$. That is to say, write down the left-hand side of (1) for $n=2$, and verify that it agrees with the right-hand side.

(b) Check the statement $P(3)$ similarly.

(c) Now use the Principle of Mathematical Induction to prove $P(n)$ for $n \geqslant 1$. The calculations in (a) and (b) should help you in understanding the step

$$P(k) \implies P(k+1).$$

Hint: If you continue to have difficulty in spotting the pattern, then write down the statements $P(4), P(5)$ etc and stare at them.

Q2. Let $P(n)$ denote the following statement:

$$4^n+6n-1 \text{ is divisible by } 9.$$

(a) Verify that the statements $P(3)$ and $P(4)$ are true.

(b) Now use the Principle of Mathematical Induction to prove $P(n)$ for $n \geqslant 1$.

Q3. Find the sum

$$S = \sum_{m=7}^{23} (m-1)(m^2+5).$$

Your answer should be a single integer. You will need the summation formulae on page 10 of the course notes. (Answer check: if you do this correctly, then the digits in your final answer should add up to 26.)

Q4.

(a) Convert the following complex number into Cartesian form:

$$w = \frac{(\overline{1-i})^3}{3+2\,i} + \frac{1}{1+1/i}\,.$$

(b) Verify that

$$|w| = \frac{3}{26}\sqrt{130}.$$

If this does not work out, then it means that you have made a mistake in part (a).

Q5. Find all the complex roots of the following equation:

$$z^5 + (1 - i\,\frac{\sqrt{3}}{2}) = 1/2.$$

Express your answers in exponential form using the principal values of the argument.

Q6. There are exactly two complex numbers z which simultaneously satisfy both of the following equations:

$$|z+1+3\,i| = \sqrt{34}, \qquad (1-3\,i)\,z + (1+3\,i)\,\bar{z} = 4. \tag{2}$$

Find them. Hint: substitute $z = x + i\,y$ into the equations and solve.

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