1210 Assignment 2 due Monday October 28

1. Consider the polynomial equation

$$P(x) = 2x^5 + x^4 - 22x^3 + 13x^2 + 20x + 4 = 0.$$

(a) Use Descartes' rules of sign to state the number of possible positive and negative roots of the equation.
(b) Find an upper bound for $|x|$ when x is a root of the equation.
(c) Use the rational root theorem to list possible rational roots of the equation.
(d) Find all roots of the equation.

2. You are given that $x = 2 - 3i$ is a zero of the polynomial

$$P(x) = 2x^4 - 5x^3 + 21x^2 + 11x + 91.$$

Find all other zeros.

3. Prove or disprove that for any two $n \times n$ matrices A and B,

$$(A - B)(A + B) = A^2 - B^2.$$

4. If $\mathbf{u} = \langle 2, -4, 1\rangle$, $\mathbf{v} = \langle 4, -3, -2\rangle$, and $\mathbf{w} = \langle 4, 1, -5\rangle$, calculate (a) $\mathbf{u} \cdot \mathbf{v} \times \mathbf{w}$, called the scalar triple product, and (b) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$, called a vector triple product.

5. Find all vectors that are perpendicular to $\langle 1, -2, 5\rangle$, have y-components equal to 3 times their x-components, and have length 5.

6. Find parametric and symmetric equations for the line

$$x - y + 2z = 4, \quad 3x + y - z = 7.$$

In problems 7–8, find out whether there exists a plane containing the two given lines. If there is such a plane, find its equation.

7.

$$L_1 : \quad \begin{aligned} x &= 2 - t, \\ y &= 3 + 2t, \\ z &= 4 + t \end{aligned} \qquad\qquad L_2 : \quad \begin{aligned} x &= 1 + s, \\ y &= 5 - 2s, \\ z &= 5 + s \end{aligned}$$

8.

$$L_1 : \quad \begin{aligned} x &= 1 + t, \\ y &= 2 - t, \\ z &= -3 + 2t \end{aligned} \qquad\qquad L_2 : \quad \begin{aligned} x + 2y + z &= 4, \\ x - y + 2z &= -3 \end{aligned}$$