UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: December 10th, 2019, 6 - 8 pm.
DURATION: 120 minutes EXAMINERS: Jaydeep Chipalkatti and Donald Trim

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LAST & FIRST NAME: (as in TRUMP, DONALD) ______________________________

Student Number: ______________________________

I acknowledge that cheating is a very serious offence.

Signature: ______________________________

(*In Ink*)

Your university email address: ______________________________

INSTRUCTIONS

I. Please write **clearly**. You must **show your work** in adequate detail in order to get marks.

II. This exam has a title page, 19 pages including this cover page. Please check that you have all the pages.

III. The marks for each question are indicated in the left-hand margin beside the statement of the question. The total number of marks for the entire test is 90.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but indicate clearly that your work is continued.

V. ***No*** calculators, texts, notes, cell phones, pagers, translators or other electronics are permitted. No outside paper is permitted.

VI. If the QR codes on your test paper are deliberately defaced, your test will not be marked.

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[8] 1. Use the principle of mathematical induction to prove that

$$\left(\frac{3}{2}\right)^n \geqslant 1 + \frac{n}{2}, \qquad \text{for } n \geqslant 1.$$

[4] 2. Consider the polynomial

$$P(x) = 100\,x^4 + 32\,x^3 - 26\,x^2 + r\,x - 1,$$

where r is a real number. It is given that the roots of $P(x)$ are:

$$0.56, \quad -0.15, \quad -0.26, \quad -0.47.$$

Is r positive, negative or zero? You must give adequate justification for your answer.

This page may be used for continuing your work.

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[8] 3. Let z be a complex number with the property that

$$|z| + 1 = |z + 1|.$$

Show that z must be a real number. Hint: Write $z = x + i\,y$, and then compare both sides.

This page may be used for continuing your work.

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[10] 4. Consider the vectors

$$\mathbf{u} = \langle -1, 2, 5 \rangle \quad \text{and} \quad \mathbf{v} = \langle 2, 0, -3 \rangle.$$

Let θ denote the angle between $\mathbf{u}$ and $\mathbf{v}$.

(a) Find $\cos\theta$ using the dot product of $\mathbf{u}$ and $\mathbf{v}$.

(b) Find $\sin\theta$ using the cross product of $\mathbf{u}$ and $\mathbf{v}$.

(c) Now use a direct calculation to verify that

$$\cos^2\theta + \sin^2\theta = 1.$$

If this does not work out, then you have made a mistake somewhere.

This page may be used for continuing your work.

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[10] 5. Consider the vectors

$$\mathbf{u} = \langle 2, 1, -1, 3 \rangle, \qquad \mathbf{v} = \langle 1, -5, 4, 6 \rangle, \qquad \mathbf{w} = \langle 2, 23, -19, -15 \rangle.$$

(a) Show that the vectors $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ are linearly dependent.

(b) Express $\mathbf{u}$ as a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

This page may be used for continuing your work.

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[12] 6. Consider the matrix

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 1 & k \\ -3 & 0 & 1 \end{bmatrix}.$$

It is given to you that the entry in the second row and first column of A^{-1} is $-15/53$. Find the value of k. If you do this correctly, then k should come out to be a negative integer. Hint: Use the adjoint formula.

This page may be used for continuing your work.

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[10] 7. Consider the following homogeneous system of equations in the variables x, y, z, w.

$$-3\,x + 9\,y - 4\,z + w = 0, \quad 2\,x - 6\,y + z + 6\,w = 0, \quad -x + 3\,y - 5\,w = 0.$$

Notice that there is no z in the third equation. Solve the system using Gauss-Jordan elimination, and write down the basic solutions.

This page may be used for continuing your work.

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[8] 8. Let $A = \begin{bmatrix} -2 & 4 & 5 \\ 1 & -2 & -8 \\ 3 & -7 & -11 \end{bmatrix}$.

(a) Find $\det(A)$ by a cofactor expansion along the second column of A.

(b) Now find $\det(A)$ by using a sequence of row-operations to convert A into an upper triangular matrix.

Of course, your answers in (a) and (b) should agree with each other.

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[10] 9. Consider the linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ given by the formula

$$T\langle v_1, v_2, v_3\rangle = \langle 2\,v_1 - v_2 + v_3,\ v_1 - 4\,v_3,\ 3\,v_1 - v_2 - 3\,v_3\rangle.$$

(a) Write down the matrix corresponding to T.

(b) Find the image of the vector $\langle 1, -6, 3\rangle$ under T.

(c) Find a **nonzero** vector $\mathbf{v}$ in $\mathbb{R}^3$ such that $T(\mathbf{v}) = 0$.

This page may be used for continuing your work.

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[10] 10. Let $A = \begin{bmatrix} -7 & 0 & 8 \\ 0 & -2 & 0 \\ -4 & 0 & 1 \end{bmatrix}$.

(a) Show that A has one real and two complex eigenvalues by finding them explicitly.

(b) Find an eigenvector corresponding to the real eigenvalue.