UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: 31 October, 2019, 5:45 pm - 7 pm
DURATION: 75 minutes EXAMINER: Jaydeep Chipalkatti and Donald Trim

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Name: ______________________________

Student Number: ______________________________

I understand that cheating is a serious offence:

Signature: ______________________________

(*In Ink*)

INSTRUCTIONS

I. This exam has a title page, 12 pages including this cover page. Please check that you have all the pages.

II. The marks for each question are indicated in the left-hand margin beside the statement of the question. The total number of marks for the entire test is 50.

III. **Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

IV. ***No*** calculators, texts, notes, cell phones, pagers, translators or other electronics are permitted. No outside paper is permitted.

V. If the QR codes on your test paper are deliberately defaced, your test will not be marked.

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[8] 1. Use the Principle of Mathematical Induction to prove the equality

$$\frac{1}{1.\,2}+\frac{1}{2.\,3}+\frac{1}{3.\,4}+\cdots+\frac{1}{n\,(n+1)}=\frac{n}{n+1}$$

for all $n \geqslant 1$.

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[4] 2. Write the sum

$$\frac{3}{2^7} - \frac{4}{2^8} + \frac{5}{2^9} - \frac{6}{2^{10}} + \cdots + \frac{11}{2^{15}}$$

in Σ-notation. Your index of summation should start from 1.

[7] 3. Convert the complex number

$$\frac{\overline{1+i}}{\left(\sqrt{2} + e^{\frac{3\pi\, i}{4}}\right)^6}$$

into Cartesian form.

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[7] 4. Consider the polynomial

$$f(x) = x^3 + a\,x^2 + b\,x + c,$$

where a, b, c are real numbers. It is given to you that $3 - i$ and 2 are roots (= zeros) of $f(x)$. Find the values of a, b, c.

[5] 5. Consider the matrix $A = \begin{bmatrix} 3 & -1 \\ 2 & -5 \end{bmatrix}$, and let I_2 denote the 2×2 identity matrix. Now find the matrix

$$A^2 + 2\,A - 13\,I_2.$$

(Hint: If you do the calculation correctly, then the final answer is very simple.)

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[4] 6. Let I_n denote the $n \times n$ identity matrix. Let A and B be matrices such that the expression

$$B^T I_3 A - 5 I_4$$

is defined. Find the sizes of A and B.

[6] 7. Consider the vectors

$$\mathbf{u} = \langle 4, a, 7 \rangle \quad \text{and} \quad \mathbf{v} = \langle a, a-1, -4 \rangle,$$

in $\mathbb{R}^3$. It is given to you that $\mathbf{u}$ and $\mathbf{v}$ are perpendicular (i.e., orthogonal). Find all possible values of a.

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[9] 8. Consider the two lines

$$L_1 : \quad x = 2 - t, \quad y = 3 + t, \quad z = -4 - 6\,t,$$

and

$$L_2 : \quad x = s, \quad y = 2 - s, \quad z = 3 + 6\,s$$

in $\mathbb{R}^3$.

(a) Show that L_1 and L_2 are parallel. (That is to say, give a complete mathematical reasoning.)

(b) Now find the equation of the unique plane which contains both L_1 and L_2. (It may be helpful to draw a diagram.)

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