MATH 1210 (FALL 2020)
ASSIGNMENT ONE

Due Date and Time: Tuesday, October 13, 9:00 am

- Please write very clearly. You can either type-set your assignment or handwrite it. What matters is that it be clearly readable.
- Your submission must be accompanied by the **Honesty Declaration** to be found on the course web-page. You can print the declaration, fill it in and take a picture, or you can simply copy the text of the declaration and sign it.
- Please make sure that you submit a single PDF file in the dropbox in UMLearn before the due date and time. I **will not** accept a collection of several files (whether they are jpeg or pdf of whatever). If you fail to obey this instruction, then you will be given a zero on the assignment.
- This assignment carries 40 marks, which will be rescaled to 10% at the end of the term.
- Please remember: no late assignments will be accepted, not even by a minute.

Q1. This question is about the uniqueness of the reduced row-echelon form (RREF). Consider the matrix [10]

$$M = \begin{bmatrix} 3 & -12 & 1 & 14 \\ 1 & -4 & 0 & 3 \\ 7 & -28 & 3 & 36 \end{bmatrix}.$$

(1) Begin with M. Start with the row operation $R_1/3$ followed by $R_2 - R_1$ and go on to reduce M to its RREF.

(2) Begin with M again. Now start with the row operation $R_1 \leftrightarrow R_2$ followed by $R_2 - 3\,R_1$ and go on to reduce M to its RREF.

Your final answers in (1) and (2) must be *exactly* the same. If this doesn't work out, then you know that you have made a mistake somewhere. You can use the **Linear Algebra Toolkit (LAT)** to do your row-operations, but you must clearly write down all the intermediate steps.

Q2. Consider the matrices [10]

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 7 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -8 & 5 \\ 6 & 0 & 1 \\ -4 & 2 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & -9 \\ 8 & -1 \\ 3 & 4 \end{bmatrix}.$$

- Calculate $A\,B$.
- Now multiply on the right by C to calculate $(A\,B)\,C$.
- Calculate $B\,C$.
- Now multiply on the left by A to calculate $A\,(B\,C)$.

You must get $(A\,B)\,C = A\,(B\,C)$. If you don't, then you know that you have made a mistake somewhere.

Q3. Consider the following system of equations: [10]

$$2\,x + y + z = 1, \quad x - 3\,y + 5\,z = 10, \quad -x + y - 6\,z = -1.$$

(1) Solve the system using Gauss-Jordan elimination (i.e., by converting the augmented matrix to RREF). You can use the LAT if you wish, but you must show all the intermediate steps.

(2) Now solve the same system using Cramer's rule. You will need to calculate four determinants altogether. You must show the full calculation for at least two of them (using any method); you can use the LAT for the remaining two if you wish.

Of course, your solutions in (1) and (2) must agree with each other.

Q4. Consider the matrix [10]

$$M = \begin{bmatrix} 3 & -7 & 5 \\ -4 & 11 & 9 \\ 2 & -8 & 10 \end{bmatrix}.$$

- Find $\det(M)$ by cofactor expansion along column 2. Show your work in detail.
- Find $\det(M)$ by cofactor expansion along row 3. Show your work in detail.

Of course, both answers must agree with each other.

Now calculate the same determinant by using row-operations to convert it into upper triangular form. You may need to use fractions. Again, you may use the LAT but you must show all the intermediate steps.

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