Assignment-3

Due: Thursday December 10, 2020 11:00 PM (Central Standard Time)

Assignment description

This assignment is due on December 10th at 11:00 pm.

No late assignments will be accepted under any circumstances. Please don't leave things until the last minute. You should anticipate possible technical difficulties (scanner not working, internet down or whatever) and plan ahead. You can always submit before the deadline.

You must write your answers in full detail in order to get marks. In other words, show your work. When in doubt, write more rather than less.

You are welcome to contact me if you need **clarification** about a question, but please don't ask me for hints. The assignment is partly a test of whether you can do things independently.

You should attach your completed honesty declaration to the solution of Question 1.

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (12 points)

Consider the polynomial

$$f(x) = 6\,x^5 + 25\,x^4 - 5\,x^3 - 110\,x^2 - 121\,x - 35.$$

Note that $f(x)$ has 5 roots.

(a) Use the rational roots theorem to list all possible rational roots of $f(x)$.

(b) Now use the Bounds Theorem to find the upper bound Q. Then eliminate some of the possible roots from (a).

(c) Now check by possible substitution which of the possible rational roots are actual roots.

This part is long and tedious. You can use a calculator, but you should show **at least two** sample calculations in detail. If you wish, you can also write a computer program to do the checking for you. In that case, although you don't have to include the code, you should give a brief verbal description of what you have done.

(d) Now find all the remaining roots of $f(x)$.

You should attach your completed honesty declaration to the solution of this question.

Q2 (10 points)

Consider the polynomial

$$f(x) = x^5 - 10\,x^4 + 27\,x^3 + 24\,x^2 - 98\,x + 116.$$

It is given to you that $1 - i$ and $5 + 2\,i$ are two of the roots of $f(x)$. Find all the remaining roots.

Q3 (9 points)

Consider the statement $\mathbb{P}(n)$:

$$2^2 + 7^2 + 12^2 + 17^2 + \cdots + (5n-3)^2 = \frac{25}{3}\,n^3 - \frac{5}{2}\,n^2 - \frac{11}{6}\,n.$$

(a) Check that the statement $\mathbb{P}(4)$ is true by calculating the LHS and RHS. A calculator is permitted, but you should show your calculations in reasonable detail.

(b) Now prove the statement $\mathbb{P}(n)$ for $n \geqslant 1$, by using the Principle of Mathematical Induction.

Q4 (9 points)

Consider the statement

$$\mathbb{P}(n): \quad 5 \text{ divides } n\,(n^4+4).$$

(a) Check that the statements $\mathbb{P}(3)$ and $\mathbb{P}(7)$ are true.

(b) Now prove the statement $\mathbb{P}(n)$ for $n \geqslant 1$, by using the Principle of Mathematical Induction. You will need the binomial theorem.