## Midterm Test

Due: Wednesday November 4, 2020 7:35 PM (Central Standard Time)

## Assignment description

(1) The test is closed-book and closed notes.
(2) You must show your work in detail in order to get marks.
(3) You cannot use the LAT or any other online tool. However, a calculator is permitted.
(4) You cannot communicate or seek help from any other person while writing the test. The only exception is that you may communicate with the instructor by email in order to seek clarification about a question if necessary.
(5) You must submit your answers by 7:35 pm. There is a heavy penalty (10\% of marks per minute) if you submit late.

## Submit your assignment

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

## Q1 (7 points)

Consider the $4 \times 4$ matrix

$$
M=\left[\begin{array}{rrrr}
4 & x & 1 & 3 \\
2 & -1 & 3 & -2 \\
0 & 1 & 5 & 4 \\
6 & 2 & -8 & 1
\end{array}\right]
$$

It is given to you that the cofactor $C_{4,3}=58$. Find the value of $x$.

Consider the system of equations

$$
\begin{aligned}
& 3 x+2 y=4 \\
& 5 x+4 y=10
\end{aligned}
$$

In matrix form, it can be written as

$$
\underbrace{\left[\begin{array}{ll}
3 & 2 \\
5 & 4
\end{array}\right]}_{C}\left[\begin{array}{l}
x \\
y
\end{array}\right]=\underbrace{\left[\begin{array}{c}
4 \\
10
\end{array}\right]}_{b}
$$

Find $x$ and $y$ using Cramer's rule. (No marks will be given for any other method.)
Q3 (5 points)

Consider the same system of equations as in Q2.
(a) Find $C^{-1}$ using the adjoint formula.
(b) Now find the values of $x$ and $y$ using the formula

$$
\left[\begin{array}{l}
x \\
y
\end{array}\right]=C^{-1} b
$$

Q4 (7 points)

Let

$$
M=\left[\begin{array}{rrrr}
1 & -2 & -2 & 3 \\
-3 & 6 & 1 & -14 \\
-1 & 2 & 1 & -4
\end{array}\right]
$$

Convert $M$ to RREF. You should clearly write down your row-operations and intermediate matrices. (Note: you are not allowed to use the LAT. However, a calculator is permitted.)

Q5 (6 points)

Write the complex number

$$
z=\left(l^{23}+2 l^{-34}\right) \overline{(3+5 i)}+\frac{1}{i}
$$

in Cartesian form. The 'bar' indicates complex conjugate.

Q6 (8 points)

Let

$$
z_{1}=-3+5 i, \quad \text { and } \quad z_{2}=1-8 i
$$

(a) Express the complex number
in Cartesian form.
(b) Find

$$
\left|z_{1}\right|, \quad\left|z_{2}\right|, \quad\left|\frac{z_{1}}{z_{2}}\right|
$$

and verify that

$$
\left|\frac{z_{1}}{z_{2}}\right|=\frac{\left|z_{1}\right|}{\left|z_{2}\right|}
$$

## Q7 (6 points)

It is given to you that the matrix

$$
M=\left[\begin{array}{ccc}
1 & p+2 q+1 & 0 \\
0 & 2-q & 1 \\
0 & 0 & p+5 q
\end{array}\right]
$$

is in RREF. Find the values of $p$ and $q$. Then write down the matrix obtained by substituting those values.

You must clearly express your reasoning process. Use English sentences if necessary. Also, if there is more than one solution, then you must find them all. Otherwise you will not get full marks for this question.

