**MATH 1210 Assignment 1 Fall 2021 Due date: October 4**

**Attempt all questions and show all your work. Some or all questions will be marked.**

1. **Use mathematical induction on integer $n$ to prove each of the following:**

   **(a)** $1(3) + 2(3^2) + 3(3^3) + \cdots + (n+1)(3^{n+1}) \;=\; \dfrac{1}{4}[(2n+1)(3^{n+2}) + 3]$ **for** $n \geq 1$ **;**

   **(b )** $1(4) + 2(5) + 3(6) + \cdots + 2n(2n+3) \;=\; \dfrac{2}{3}(n)(2n+1)(2n+5)$ **for** $n \geq 1$ **;**

   **(c )** $\dfrac{(2n)!}{(n!)^2} > 2^n$ **for** $n \geq 2$ **;**

   **(d )** $a^{3n} - b^{3n}$ **is divisible by** $a^2 + ab + b^2$**, where** $a$ **and** $b$ **are fixed integers and** $n$ **is any positive integer.**

2. **Write the sum in sigma notation.**

$$3 - \frac{4(5)}{\sqrt{3}} + \frac{9(7)}{\sqrt{5}} - \frac{16(9)}{\sqrt{7}} + \cdots + \frac{121(23)}{\sqrt{21}}$$

3. **Consider the sum** $(3)^2 + (8)^2 + (13)^2 + \cdots + (15n-2)^2$**:**

   **(a) Write the sum in sigma notation.**

   **(b) Use identities** $\displaystyle\sum_{k=1}^{m} k = \frac{1}{2}\,[m(m+1)]$ **and** $\displaystyle\sum_{k=1}^{m} k^2 = \frac{1}{6}\,[m(m+1)(2m+1)]$ **to prove that**

$$(3)^2 + (8)^2 + (13)^2 + \cdots + (15n-2)^2 = \frac{1}{2}(n)\,(450n^2 + 45n - 11)\,.$$

4. **Prove that** $\displaystyle\sum_{\ell=1}^{2n} \ell(\ell+1) = \frac{4}{3}\,[n(n+1)(2n+1)]$ **by each of the following two methods:**

   **(a)** **By mathematical induction on positive integer** $n \geq 1$ **.**

   **(b)** **By using the identities mentioned in part (b) of question** $3$ **.**

5. **Evaluate** $\dfrac{1}{2^{90}} z^{91} \;+\; (\overline{-i}\,)^{91} \;+\; \dfrac{-2 + \sqrt{3}i}{i}$ **, where** $z = -\sqrt{3} + i$ **. Simplify as much as possible.**

6. **For each of the following statements, if it is true prove it, and if it is false give a counter example.**

   **(a)** $\dfrac{\overline{z}}{|z|^2} = \dfrac{1}{z}\,, \quad (z \neq 0)$ **;**

   **(b)** $\arg(z + \overline{z}) = 0$ **;**

   **(c)** $\dfrac{e^{4\theta^2 i}\,(e^{\theta\, i})^4}{e^{i^7}} \;=\; \cos(2\theta+1)^2 \;+\; i\sin(2\theta+1)^2$ **.**

7. **Find all fifth roots of** $z = -16\sqrt{2} - 16\sqrt{2}\,i$ **. Write the roots in exponential form and use principal value of their arguments.**