## UNIVERSITY OF MANITOBA

Second Assignment

COURSE: MATH 1210 DATE & TIME: October 22, 5pm

This assignment is due on 5pm on October 22. Attempt all questions and show your work. Some or all questions will be marked. Simplify your answers as much as possible. If your final answer contains complex numbers, they must be given in Cartesian form, unless other instructions are given.

1. Consider the polynomial

$$P(x) = 2x^4 - 3x^3 - 4x^2 + 14x - 12.$$

- (a) This polynomial has two rational zeroes. Use the rational root theorem and Descartes' rule of signs to find them.
- (b) Use long division to find the remaining roots of P(x) = 0.
- (c) Write P(x) as a product of linear factors.
- 2. Consider the polynomial

$$P(x) = \sqrt{2}x^4 - \sqrt{8}x^2 + \sqrt{8}x^4$$

- (a) Find all roots of P(x) = 0. If your answer contains any complex numbers, they must be in exponential form. (Use the principal value of the argument.)
- (b) Write P(x) as a product of linear factors.
- 3. Consider the polynomial

$$P(x) = 9x^3 - 12x^2 - 20x + 16$$

- (a) Use the bounds theorem together with the rational root theorem to give a list of possible rational roots of P(x) = 0.
- (b) Write P(x) as a product of linear factors. You can make use of Descartes' rule of signs to make your life easier.
- 4. Let  $\theta$  and  $\phi$  be real numbers, and let A and B be the matrices

$$A = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}, \qquad B = \begin{pmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{pmatrix}.$$

(a) Show that

$$AB = \begin{pmatrix} \cos(\theta + \phi) & \sin(\theta + \phi) \\ -\sin(\theta + \phi) & \cos(\theta + \phi) \end{pmatrix}$$

You may use the identities at the end of this file. (You don't need to use all of them.)

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(b) Use induction on n to prove that, for all integers  $n \ge 1$ , we have

$$A^{n} = \begin{pmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{pmatrix}.$$

Recall that

$$A^n = \underbrace{AA \dots A}_{n \text{ times}}.$$

$$\begin{aligned} \sin(\phi \pm \theta) &= \sin(\phi)\cos(\theta) \pm \cos(\phi)\sin(\theta), & \cos(\phi \pm \theta) = \cos(\phi)\cos(\theta) \mp \sin(\phi)\sin(\theta), \\ 2\cos(\theta)\cos(\phi) &= \cos(\theta - \phi) + \cos(\theta + \phi), & 2\sin(\theta)\sin(\phi) = \cos(\theta - \phi) - \cos(\theta + \phi), \\ 2\sin(\theta)\cos(\phi) &= \sin(\theta + \phi) + \sin(\theta - \phi), & 2\cos(\theta)\sin(\phi) = \sin(\theta + \phi) - \sin(\theta - \phi). \end{aligned}$$