MATH 1210 A03 Assignment 3 Fall 2021 **Due date: January 7 before 5 PM**

Attempt all questions and show all your work. Some or all questions will be marked.

1. **Solve each of the following system of linear equations.**
   (a) **Use Gauss-Jordan elimination to solve**

$$\begin{array}{rrrrcr} 3x_1 & -9x_2 & +x_3 & +4x_4 & = & 0 \\ x_1 & -3x_2 & & +2x_4 & = & -1 \\ -x_1 & +3x_2 & & -2x_4 & = & 1 \\ 2x_1 & -6x_2 & +x_3 & +2x_4 & = & 1 \end{array}$$

   (b) **Use Gaussian elimination to solve**

$$\begin{array}{rrrrrcr} x_1 & +2x_2 & & -x_4 & +x_5 & = & -1 \\ 6x_1 & +14x_2 & +8x_3 & & +18x_5 & = & -3 \\ x_1 & +3x_2 & +4x_3 & +2x_4 & -7x_5 & = & \frac{7}{2} \end{array}$$

2. **Determine if** $\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ **is a basic solution of the following homogeneous system of linear equations. Show your work.**

$$\begin{array}{l} x_1 + 2x_2 + x_3 + 3x_4 - x_5 = 0 \\ x_1 + 2x_2 + x_3 + 4x_4 = 0 \\ 3x_1 + 6x_2 + 3x_3 + 9x_4 - 3x_5 = 0 \end{array}.$$

3. **Use Cramer's rule to find *only* the value of** x_2, **where** a **is a positive real number.**

$$\begin{array}{l} 2x_2 - x_3 + x_4 = -4 \\ 4x_3 + ax_4 = 0 \\ x_2 - 5x_3 + 2x_4 = -2 \\ x_1 + 7x_2 + 8x_3 + 6x_4 = 10 \end{array}.$$

4. **Let** $A = \begin{pmatrix} 1 & 2a & a \\ -2 & -2 & a \\ a & 0 & 0 \end{pmatrix}$ **and** $B = \begin{pmatrix} 0 & -a & -2a \\ 2 & 3 & -2a \\ 0 & 1 & a \end{pmatrix}$. **Find all values of** a **for which** $det(A+B) = 0$.

5. **Given that** $\begin{vmatrix} a & 1 & e \\ b & d & f \\ c & -1 & g \end{vmatrix} = 10$, **use properties of determinant to find** $\begin{vmatrix} b+c & d-1 & f+g \\ b+2a & d+2 & f+2e \\ 4c-a & -5 & 4g-e \end{vmatrix}$.

6. **Let** A, B **and** C **be** $n \times n$ **invertible matrices such that** $|A| = 2$, $|B| = -3$ **and** $|A+I| = 6$. **Find determinant of each of the following:**
   (a) A^2B^T;
   (b) $(C^{-1}A)^{-2}B^3C^{-2}$;
   (c) $B^{-1}(I + BA^{-1}B^{-1})BA$.

7. **Determine if each of the following set of vectors are linearly dependent or linearly independent. Show your work.**

(a) $\mathbf{u_1} = \langle 1, 2, -1 \rangle$, $\mathbf{u_2} = \langle 0, 1, 1 \rangle$, $\mathbf{u_3} = \langle 5, 1, 7 \rangle$, $\mathbf{u_4} = \langle 4, -2, 15 \rangle$;

(b) $\mathbf{u_1} = \langle 1, 2, -1 \rangle$, $\mathbf{u_2} = \langle 0, 1, 1 \rangle$, $\mathbf{u_3} = \langle 5, 1, 7 \rangle$;

(c) $\mathbf{u_1} = \langle 1, 2, -1, a \rangle$, $\mathbf{u_2} = \langle b, 0, 1, 1 \rangle$, $\mathbf{u_3} = \langle 5, 1, c, 2 \rangle$, $\mathbf{u_4} = \langle 0, 0, 0, 0 \rangle$.

8. Let $\mathbf{u} = \langle 1, 2, -1, 5 \rangle$ **,** $\mathbf{v} = \langle 0, 1, 1, 0, \rangle$ **and** $\mathbf{w} = \langle 3, 4, -2, 15 \rangle$**. Show that u ,v and w are linearly independent and write the vector** $\mathbf{a} = \langle 1, 1, 1, 5 \rangle$ **as a linear combination of u ,v and w.**

9. Let $A = \begin{pmatrix} 1 & 1 & -2 \\ 6 & -1 & 4 \\ -3 & 2 & 1 \end{pmatrix}$ **and** $B = \begin{pmatrix} 1 & 2 & 3 \\ -6 & -2 & -3 \\ 5 & -1 & -1 \end{pmatrix}$**. Find the inverse of the matrix** $A + B - I$**.**