UNIVERSITY OF MANITOBA
Mock/Practice Final

COURSE: MATH 1210 DURATION: 50 minutes
DATE & TIME: December 6, 9:30am EXAMINER: Kristel/Comicheo

This is a **practice** exam, to help prepare you for the final. Attempt all questions and show your work. Simplify your answers as much as possible. You may use your textbook, and the files available on UM learn. You may use the formulas at the end of this exam.

[6] 1. Consider the polynomial

$$P(x) = x^6 - 2\sqrt{2}x^3 + 4.$$

Find all roots of $P(x) = 0$. Give your answers in exponential form. Use the principal value for the argument.

[7] 2. Let x be a real number, and let A be the matrix

$$\begin{pmatrix} 2 & x \\ 0 & 1 \end{pmatrix}.$$

Use proof by induction to show that, for all positive integers n, we have

$$A^n = \begin{pmatrix} 2^n & (2^n - 1)x \\ 0 & 1 \end{pmatrix}.$$

[3] 3. Let A and B be the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -4 & 7 \\ 0 & 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} -3 & 0 & 0 \\ 11 & 1 & 0 \\ -12 & 5 & 2 \end{pmatrix}$$

Calculate $|AB|$.

[5] 4. Evaluate the sum

$$\sum_{j=3}^{k} 2j(3j - 14).$$

(This problem has nothing to do with induction.)

[4] 5. Let t be a real number. Calculate the determinant of

$$A = \begin{pmatrix} 1 & e^{it} & e^{-it} \\ -1 & e^{it} & e^{-it} \\ 0 & e^{-it} & e^{it} \end{pmatrix}$$

by expanding along the first column. Give your answer in Cartesian form.

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[5] 6. For each of the following matrices, find the inverse, if possible.

$$A = \begin{pmatrix} 0 & 1 & -2 \\ 0 & 8 & 1 \\ 0 & -4 & 7 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & -11 \\ 2 & 4 & 6 \\ 3 & 6 & -3 \end{pmatrix}, \qquad C = \begin{pmatrix} 6 & -2 \\ 2 & \frac{1}{2} \end{pmatrix}.$$

[6] 7. Let v and w be the vectors

$$v = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}, \qquad w = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}.$$

Find all vectors

$$u = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

such that the equation $v \times u = w$ holds.

FORMULAS:

quadratic formula: $x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

sum of integers: $\sum_{n=1}^{m} n = \frac{m(m+1)}{2}$.

sum of squares: $\sum_{n=1}^{m} n^2 = \frac{m(m+1)(2m+1)}{6}$.

sum of cubes: $\sum_{n=1}^{m} n^3 = \frac{m^2(m+1)^2}{4}$.

Euler's formula: $e^{it} = \cos(t) + i\sin(t)$.

This practice exam does not cover all the topics that will be covered on the final.