MATH 1210 Assignment 1 Fall 2022 **Due date: October 3, 5:00 PM**

Attempt all questions and show all your work. Some or all questions will be marked.

1. Use mathematical induction on integer n to prove each of the following statements.

   (a) $-1 \cdot 3 + 2 \cdot 5 - 3 \cdot 7 + 4 \cdot 9 - \ldots + (2n)(4n+1) \;=\; n(4n+3)$ for $n \geq 1$;

   (b) $\displaystyle\sum_{j=1}^{n} \frac{(j-1) \cdot j!}{2^j} \;=\; \frac{(n+1)!}{2^n} - 1$ for $n \geq 1$;

   (c) $3^n + 2n^2 - 1$ is divisible by 4 for $n \geq 3$;

   (d) Let $x \geq 1$ be a fixed real number. Prove that $(2x-1)^n \geq 2x^n - 1$ for $n \geq 1$.

2. Write the following sum in sigma notation.

$$\frac{21 \cdot 5}{2} - \frac{19 \cdot 9}{4} + \frac{17 \cdot 13}{8} - \frac{15 \cdot 17}{16} + \ldots + \frac{5 \cdot 37}{512}$$

3. Use the identities

$$\sum_{k=1}^{m} k = \frac{1}{2}\,[m(m+1)], \qquad \sum_{k=1}^{m} k^2 = \frac{1}{6}\,[m(m+1)(2m+1)], \qquad \sum_{k=1}^{m} k^3 = \frac{1}{4}\,[m^2(m+1)^2]$$

   to evaluate the sum

$$\sum_{k=-100}^{100} \big((k+101)^3 - 3k - 305\big).$$

   You do not need to simplify your final answer to a single number: for example, expressions like $\dfrac{51 \cdot 52 \cdot 103}{6}$ do not require further simplification.

4. Consider the sum $n(n+2) + (n+1)(n+3) + (n+2)(n+4) + \ldots + 3n(3n+2)$.

   (a) Write the sum in sigma notation.

   (b) Use the identities from question 3 to prove that for $n \geq 2$

$$n(n+2) + (n+1)(n+3) + (n+2)(n+4) + \ldots + 3n(3n+2) \;=\; \frac{13n(2n^2+3n+1)}{3}\,.$$

5. Write the following complex numbers in Cartesian from. Simplify as much as possible.

   (a) $\dfrac{(2-3i)(3+5i)}{1+\sqrt{2}i}$

   (b) $\dfrac{\overline{3e^{7\pi i/20}} \cdot i^{27}}{4e^{22\pi i/5}}$

   (c) $\dfrac{\left(-\sqrt{2}+\sqrt{6}i\right)^{10}}{(1-i)^{26}}$

6. Find all complex solutions of the equation $8z^4 - \sqrt{3} - i = 0$. Write the roots in exponential form and use principal value of their arguments.